Targeted Persuasion

Gabriele Gratton Christopher Teh DJ Thornton*

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Abstract

Sender maximizes how many receivers buy a widget of uncertain quality. She targets receivers to privately communicate information about the widget's quality. After the target chooses whether to buy the widget, other receivers observe the target's choice with probability increasing in the target's popularity. We prove two results that hold remarkably generally: Sender optimally communicates with targets as if no other receiver exists; a target's popularity is a double-edged sword for Sender. We fully characterize the optimal choice of a single target under multiple communication protocols and establish that Sender can benefit from protocols that constrain her communication.

*Gratton: UNSW Business School, UNSW Sydney. Email: g.gratton@unsw.edu.au. Teh: UNSW Business School, UNSW Sydney. Email: chris.teh@unsw.edu.au. Thornton: UNSW Business School, UNSW Sydney. Email: d.thornton@unsw.edu.au. We thank Anton Kolotilin, Barton E. Lee, Clement Minaudier, and Nageeb Ali for useful feedback. Gratton is the recipient of an Australian Research Council Future Fellowship (FT210100176) and his research was supported under the Australian Research Council's Discovery Projects funding scheme (Project DP240103257).

Verba movent, exempla trahunt.
(Words persuade, examples compel.)

Latin proverb

1 Introduction

Startup entrepreneurs aim to persuade many investors to fund their startups. But explaining their business idea often requires complex explanations and lengthy demonstrations. As a result, they are limited to privately communicate only with a handful of potential investors they have targeted in their networking activities and with whom they have built stronger connections. Yet, they may reasonably hope that, were they to persuade their target investors to invest in their startup, their example will compel others to do the same. Therefore, an entrepreneur needs to choose both *who to target* in her networking activities, as well as *how to persuade them*. Because she communicates with her target privately, whether and how the target's example may influence others naturally depends on others' expectations about what she may have told the target. Thus, intuitively, the entrepreneur's choice of communication should maximize, other than the probability of persuading the target to invest, also the power of the target's example over other potential investors. In turn, the entrepreneur's choice of target depends on her anticipation of the target's influence over other potential investors. The entrepreneur may, for instance, choose to target investors who are more likely to be fans of her ideas, increasing the chance they may be persuaded. Or, on the contrary, the entrepreneur may target famously skeptical targets, in the hope that their example may be more compelling for others. Finally, the entrepreneur's choice of target may take into account that an example set by a more popular target is likely to be observed by—and hopefully compel to invest—a greater number of other investors.

This dual problem of who to target and how to persuade them is common to many markets and organizations, from marketing strategies aimed at targeting "influencers" to adopt a new product or technology, to lobbying efforts aimed at popular policymakers whose example may compel others to vote for a new law or adopt a new policy. In fact, this problem may be more common in the era of social media, when many users can observe the example set by early adopters of a new product, technology, or policy position, but can only indirectly infer what these influencers knew when they made their choice.

In all these situations, (i) an interested party can choose who to target in networking activities and (ii) what and how to communicate to her target. Crucially, (iii) she cares about persuading a larger number of decision-makers, but non-targeted decision-makers can only observe actions taken by the target (and not what was communicated). In this paper we introduce a one-Sender, many-receivers model of these problems of targeted persuasion. In our benchmark model of Section 2, (i) Sender chooses a target receiver and (ii) privately communicates with the target about the quality of a widget; (iii) Sender's objective is to convince as many receivers as possible to buy the widget, but non-targeted receivers can only observe the target's positive or negative *example*: his choice of whether to buy or not. Our benchmark model and key results do not depend on the specific communication protocol available to Sender. Furthermore, in Section 5 we generalize the model in several directions, allowing for more general utility functions of Sender and for the receivers' choices to be strategic complements, for Sender to choose multiple targets, for other receivers not to observe Sender's choice of target but only positive examples, and for other receivers to observe examples set by non-targeted receivers. Finally, we discuss how the central message of our results does not change if Sender is informed about the quality of the widget when she chooses her target.

In our model, receivers differ in two dimensions. First, some are more *skeptical* than others, in the sense that they buy the widget only when they have greater beliefs that the widget is good. In fact, some are *fans*: they would buy the widget in the absence of any further information. Second, some are more *popular* than others, in the sense that other receivers observe their choice with greater probability. As we discuss later, a receiver's popularity naturally maps into well-known concepts of network centrality.

In Section 3, we study Sender's problem of *how to persuade* a chosen target. Because of her objective, Sender would ideally credibly induce other receivers to believe she communicates with the target in the way that maximizes the compelling power of the target's example. However, in equilibrium, other receivers correctly anticipate that, in the privacy of the Sender-target communication, Sender would renege on such a commitment and simply try to persuade the target so as to maximize the probability that he will set a positive example. Therefore, any Sender's plan to fool other receivers will "unravel." This logic leads to our *Example-unraveling theorem*. It says that, in any equilibrium, and independently of the choice of target, Sender's optimal private communication maximizes the probability that the target buys the widget. That is, Sender optimally communicates with the target *as if* no other receiver exists, even though Sender's objective is, in fact, to persuade as many receivers as possible.

In Section 4, we exploit this feature of our model to derive two general results regarding the optimal choice of *who to target*. First, we show that all equilibria are of one of two types. In *only fans equilibria*, Sender optimally chooses a target and communicates with him in such a way that the target buys the widget if and only if he is a fan. Thus, in equilibrium, the target's example carries no information for other receivers, and they too buy the widget if and only if they are fans. However, this does not mean that Sender communicates to the target no useful information for other receivers—rather, that this information is not sufficient to affect the target's choice and so his example has no power to convey this information to other receivers. In contrast, in *informative example equilibria*, Sender's optimal choice of target and communication induces the target to buy with probability strictly between 0 and 1. Thus, in equilibrium, the target's example has the power to influence other receivers. In particular, a positive example will (generically) compel some skeptical receivers to buy and a negative example will compel some fans not to buy. This feature of equilibrium behavior underpins the second result: what makes a target valuable for Sender crucially depends on his popularity in both positive and negative ways. In fact, an increase in a target's *popularity is a double-edged sword* for Sender. On the one hand, a more popular target is more likely, via a positive example, to persuade other receivers to buy the widget. On the other hand, he is also more likely to persuade them, via a negative example, *not* to buy the widget.

We show in Section 6 how these results allow us to fully characterize equilibrium behavior under different communication protocols. We focus on two canonical cases: *Bayesian persuasion* à la Kamenica and Gentzkow (2011) and (generalized) *information disclosure* (Dye, 1985; Milgrom, 1981). When Sender's communication with the target takes the form of Bayesian persuasion, Sender optimally targets a skeptical receiver only if he is not too skeptical. Intuitively, Sender anticipates that targeting a skeptical receiver entails a tradeoff: if the target is persuaded to buy, his positive example will compel all receivers less skeptical than he is to buy the widget; but if he is not persuaded, his negative example will compel all receivers, including all fans, *not* to buy the widget. Because the probability to persuade a target is decreasing in the target's skepticism, some targets are just too skeptical for Sender. If all skeptical receivers are too skeptical, an only fans equilibrium arises in which Sender gives up on the idea of persuading anybody other than fans who are already willing to buy the widget.

What makes a skeptic "too skeptical" for Sender crucially depends on the double-edged sword of his popularity. Intuitively, the positive effect of popularity dominates when the target is less skeptical, when there are more skeptical receivers who are not as skeptical as he is, or when there are fewer fans. We show that in fact the negative effect of popularity may dominate for Sender's equilibrium choice of a skeptical target. I.e., in equilibrium, a marginal increase in the target's popularity may induce Sender to optimally switch to a less popular target, or even to targeting a fan (thus inducing an only fans equilibrium), to avoid losing too many fans in the event of a negative example.

We compare these results with the case when Sender-target communication takes the form of information disclosure. We characterize how Sender optimally communicates with her target and how other receivers interpret the target's example in Sender's preferred equilibrium. Importantly, we show that the less freedom afforded to Sender to "tailor" her communication to the target under information disclosure may in fact benefit Sender. Intuitively, recall that in our setting Sender would ideally credibly induce other receivers to believe she communicates with the target in the way that would maximize the power of the target's example. However, our Example-unraveling theorem says that such a commitment is not credible. Under Bayesian persuasion this forces Sender to choose the target based on the target's skepticism, because the target's positive example can only affect less skeptical receivers. In contrast, under information disclosure Sender is in part (mechanically) committed to how she communicates with the target. Therefore, the effect of a target's positive example is in part independent of the target's skepticism and his example may affect even receivers more skeptical than he is. The lesser freedom in choosing how to communicate affords Sender greater freedom in choosing the optimal target based on his popularity, in turn increasing the probability that the target's example compels more receivers. Thus, when popularity has a positive effect on a target's value, Sender's expected payoff may be strictly greater under information disclosure than under Bayesian persuasion.

Our model includes ingredients from both the Sender-Receiver communication literature and models of observational learning. A key ingredient is that the action of the target is observed by other receivers and, because the target in equilibrium possesses greater information, his example may compel other receivers. We share this observational structure with models of social learning (e.g., Banerjee, 1992; Bikhchandani et al., 1992) and our specific one-to-many process is reminiscent of the classic two-step flow model of Katz and Lazarsfeld (1955). This simple observational structure allows us to analyze how the target's popularity affects the Sender's choice of who to target. Our notion of popularity can be interpreted as an agent's diffusion centrality (Banerjee et al., 2013) in a larger "network" among receivers, i.e., a measure of how many other agents are likely to observe the target's action in the presence of exogenous and independent link failures (see Appendix E for details). We think of our reduced-form approach as capturing a feature of many real-world networks in which agents lack complete information about the network structure (who "follows" who), but can rely on proxies that measure influence, such as follower counts or ratings.

Works that combine social learning (or social experimentation[1]) with the presence of a sender include Caminal and Vives (1996) and Welch (1992). Arieli et al. (2023) study optimal information design to persuade a sequence of receivers who observe predecessors' actions. Recent work on Bayesian persuasion in networks (Candogan et al., 2020; Kerman and Tenev, 2021) emphasizes how network topology shapes the optimal information structure. However, these models do not tackle the question of who Sender should target to start such a process of social learning.

Similarly, Arieli and Babichenko (2019), Bardhi and Guo (2018), Chan et al. (2019), and Wang (2015) study private Bayesian persuasion of a group. However, in their context, Sender can communicate with all receivers. Therefore, these models are not well suited to understand who Sender should target to communicate with or how she should communicate with the target when his example may persuade others.

Our focus on who Sender should target is shared by an extensive literature on targeting "influencers" in networks (e.g., Ballester et al., 2006; Galeotti and Goyal, 2009). This literature often abstracts away from the question of how Sender should persuade by assuming that targets take the desired action. In contrast, we capture influence in a reduced form but allow the target's example to be endogenous in the sense that Sender needs to persuade the target to buy.

Caillaud and Tirole (2007) and Schnakenberg (2017) combine the question of who Sender should communicate to with the idea that the target of this communication may in turn talk to other receivers. Caillaud and Tirole (2007) study a model in which Sender chooses who to target and the target's example can persuade others. However, in their model Sender cannot choose how to communicate to the target and other receivers do not need to make conjectures about what type of information Sender communicated. Schnakenberg (2017) models lobbying through cheap talk (Crawford and Sobel, 1982) with heterogeneous legislators, showing how Sender can target allied legislators to act as intermediaries. Awad (2020) and Awad and Minaudier (2025) instead allow Sender to communicate hard information. In these models, targeted legislators choose whether to relay Sender's message or endorse a policy via cheap talk.[2] In contrast, in our model indirect persuasion occurs through the action (example) of the target and we study how different receivers' skepticism and their popularity among other receivers combine in de-

[1] In models of social experimentation, agents observe *outcomes* of other agents rather than the *actions* they take, so there is an informational externality but no informational asymmetry (Gale and Kariv, 2003).

[2] In these political applications, Sender wishes to persuade a sufficiently large set of receivers—typically, a majority. In our model, Sender wishes to maximize the number of receivers who are persuaded. Nevertheless, as we discuss in Section 5, our central results only rely on the assumption that Sender's payoff is increasing in the expected number of receivers who are persuaded and therefore applies to majority settings if voting is probabilistic (Lindbeck and Weibull, 1987).

termining Sender's optimal choice of target.

Egorov and Sonin (2019) also study a problem in which Sender wishes to persuade many receivers and receivers may indirectly learn Sender's message from other receivers. However, their focus is on who Sender should "attract" in designing messages that receivers can only access at a cost. Other receivers are aware of how Sender communicates, but may not be willing to pay the cost of observing the actual message. In contrast, in our model Sender can choose who to target directly and then can choose how to communicate with him, while the problem for other receivers is that they do not necessarily know how and what Sender communicated.

2 A model of targeted persuasion

We begin with a benchmark model in which: (i) Sender's payoff is equal to the number of receivers who buy the widget and receivers' payoffs depend only on their own choice and the quality of the widget, but not on other receivers' choices; (ii) Sender's target is a single receiver; (iii) non-targeted receivers observe the identity of the target (but may not observe his choice); (iv) non-targeted receivers can observe the choice of the target, but not of other receivers; and (v) at the time of choosing the target, Sender is uninformed about the quality of the widget. Section 5 relaxes each of these assumptions and shows that our main results continue to hold.

2.1 Benchmark model

There is a Sender ("she") and $R \geq 2$ receivers ("he"), indexed by $r \in \mathcal{R} \equiv \{1, \dots, R\}$.

Sender. Sender wishes to maximize the number of receivers who buy a widget of uncertain quality, $\theta \in \{G, B\}$. The widget is *good* ($\theta = G$) with probability $\mu \in (0, 1)$. Otherwise it is *bad* ($\theta = B$). Let $a_r = 1$ if receiver $r \in \mathcal{R}$ buys the widget and $a_r = 0$ otherwise. Sender maximizes $\sum_{r \in \mathcal{R}} a_r$.

Receivers. Each receiver has a unit demand for the widget and buys the widget if and only if he believes it is good with sufficiently high probability. Formally, let p_r be receiver r's (posterior) belief that the widget is good. Receiver r buys the widget if and only if $p_r \geq \sigma_r$, where $\sigma_r \in [0, 1]$ is receiver r's publicly known *skepticism*. Without loss of generality, we order receivers by their skepticism: $\sigma_1 \leq \sigma_2 \leq \dots \leq \sigma_R$. We say that receiver r is a *fan* if $\sigma_r \leq \mu$, i.e., he chooses to buy in the absence of any further information;

otherwise, he is a *skeptic*. Let $F \equiv \max\{r \in \mathcal{R} : \sigma_r \leq \mu\}$ be the number of fans. To avoid uninteresting cases, we assume that there is at least one fan and at least one skeptic: $\sigma_1 \leq \mu < \sigma_R$.

Targeted persuasion. A targeted persuasion game plays out as follows. First, Sender chooses a *target* $t \in \mathcal{R}$. Second, nature chooses quality $\theta \in \{G, B\}$. Third, Sender privately communicates with t. We discuss below what information Sender may obtain about the widget and how she may communicate it to the target. Fourth, t purchases the widget if and only if $p_t \geq \sigma_t$ and nature determines whether each non-targeted receiver observes t's choice: each receiver $r \neq t$ observes receiver t's choice with probability equal to t's publicly known *popularity*, $\pi_t \in (0, 1)$. Finally, each receiver r buys the widget if and only if $p_r \geq \sigma_r$.

Private communication. Communication between Sender and her target is private. The specific *communication protocol* specifies how the interaction between Sender and target affects: (i) what information Sender obtains about the widget and (ii) how it is communicated to her target. We model a generic communication protocol as a publicly known triple $\mathcal{CP} = (M, A, I)$, where M are the possible messages privately observed by Sender and 2^M is the set of possible messages the target may observe from Sender's communication. $A(m_S) \subseteq 2^M$ is the set of *allowable* messages that Sender may communicate to the target, conditional on observing $m_S \in M$. I is a collection of *information structures*, i.e., families of conditional distributions over M of the form $i = \{i(\cdot \mid \theta)\}_{\theta \in \{G,B\}}$.

A communication protocol $\mathcal{CP}$ induces a communication game between Sender and the target as follows. First, Sender chooses an information structure $i \in I$, observed by the target. Conditional on θ, Sender privately observes a message m_S drawn with probability $i(m_S \mid \theta)$. Finally, Sender chooses which allowable message $m_t \in A(m_S)$ to communicate to the target. Notice that the communication between Sender and the target is private so that non-targeted receivers do not observe Sender's choice of information structure, nor do they observe which message Sender chooses to communicate to the target.

Our generic communication protocol encompasses many examples of interest, including two canonical cases we study in greater detail in Section 4. First, when I includes all possible information structures and $A(m_S) = \{m_S\}$ for all $m_S \in M$, so that Sender commits to truthfully communicate the message received, the communication game is one of *Bayesian persuasion* (Kamenica and Gentzkow, 2011). Second, when I is a singleton, so that Sender is simply endowed with information, and $A(m_S) \equiv \{m_t \in 2^M : m_S \in m_t\}$, so that Sender cannot lie about the message received, the communication game is one of *information disclosure* à la Milgrom (1981).[3]

Solution concept. A strategy for Sender is a triple $(t, \{i_t\}_{t\in\mathcal{R}}, \{c_t\}_{t\in\mathcal{R}})$, where t is the Sender's choice of target, and, for each possible t, i_t is Sender's choice of information structure, and

$$c_t \in C \equiv \{c : M \to \Delta(2^M) \mid \forall m_S \in M,\ \text{supp}(c(\cdot|m_S)) \subseteq A(m_S)\}$$

is Sender's communication strategy for each possible observed message m_S.

For each receiver $r \in \mathcal{R}$, if he is the chosen target, t, his posterior $p_t(i_t, m_t)$ is a function of both Sender's choice of information structure i_t and the observed message m_t; if he is not the target, his posterior $p_r(t \mid o_t)$ for each possible target $t \neq r$ is a function of the target's identity t, and r's (private and independent) observation $o_t \in \{0, 1, \emptyset\}$ of the target's action, where $o_t = 0$ when r observes that t does not buy, $o_t = 1$ when he observes that t buys, and $o_t = \emptyset$ when he does not observe t's action.[4]

It is useful to define Sender's expected payoff (the number of widgets sold) given a choice of target t, $V_t(m_t, p_t, \boldsymbol{p}_{-t})$, as a function of the message communicated to the target, m_t, the target t's posterior belief, p_t, and the non-targeted receivers' posterior beliefs when Sender targets t, $\boldsymbol{p}_{-t} \equiv \{p_r(t)\}_{r\neq t}$:

$$\begin{aligned} V_t(m_t, p_t, \boldsymbol{p}_{-t}) \equiv \mathbb{1}[p_t(i_t, m_t) \geq \sigma_t] \left(1 + \sum_{r\neq t} \left(\pi_t \mathbb{1}[p_r(t \mid 1) \geq \sigma_r] + (1-\pi_t)\mathbb{1}[p_r(t \mid \emptyset) \geq \sigma_r)\right]\right) \\ + \mathbb{1}[p_t(i_t, m_t) < \sigma_t] \left(\sum_{r\neq t} \left(\pi_t \mathbb{1}[p_r(t \mid 0) \geq \sigma_r] + (1-\pi_t)\mathbb{1}[p_r(t \mid \emptyset) \geq \sigma_r)\right]\right) \end{aligned}$$

In what follows, we characterize the set of perfect Bayesian equilibria (Fudenberg and Tirole, 1991)—henceforth "equilibrium." In our context, an assessment $(t, \{i_t\}_{t\in\mathcal{R}}, \{c_t\}_{t\in\mathcal{R}}, \{p_t, \boldsymbol{p}_{-t}\}_{t\in\mathcal{R}})$ is an equilibrium if:

1. For each target $t \in \mathcal{R}$, the posterior $p_r(t)$ of each non-targeted receiver $r \neq t$ is

[3] We allow for Sender to be partially, fully, or not informed, and for her information endowment to be unknown to the target. This accommodates other well-known extensions of Milgrom's setting, e.g., in Dye (1985) and Hart et al. (2017).

[4] We abuse notation slightly. Formally, for each possible target t and non-targeted receiver r's observation o_t, in equilibrium a target's posterior p_t equals the value of the function $p_t(i_t, m_t)$; a non-targeted receiver's posterior p_r equals the value of the function $p_r(t \mid o_t)$.

derived using Bayes' rule, Sender's strategy, and the target's posterior, p_t: [5]

$$p_r(t \mid 0) = \frac{\sum_{\substack{m_t \in A(m_S):\\ p_t(i_t, m_t) < \sigma_t}} \mu \sum_{m_S \in M} i_t(m_S|\theta = G) c_t(m_t \mid m_S)}{\sum_{\substack{m_t \in A(m_S):\\ p_t(i_t, m_t) < \sigma_t}} \sum_{\theta \in \{G,B\}} \Pr(\theta) \sum_{m_S \in M} i_t(m_S|\theta) c_t(m_t \mid m_S)}; \tag{1}$$

$$p_r(t \mid 1) = \frac{\sum_{\substack{m_t \in A(m_S):\\ p_t(i_t, m_t) \geq \sigma_t}} \mu \sum_{m_S \in M} i_t(m_S|\theta = G) c_t(m_t \mid m_S)}{\sum_{\substack{m_t \in A(m_S):\\ p_t(i_t, m_t) \geq \sigma_t}} \sum_{\theta \in \{G,B\}} \Pr(\theta) \sum_{m_S \in M} i_t(m_S|\theta) c_t(m_t \mid m_S)}; \tag{2}$$

$$p_r(t \mid \emptyset) = \mu. \tag{3}$$

2. For each target $t \in \mathcal{R}$, his posterior p_t is derived using Bayes' rule and Sender's strategy: for all m_t Sender communicates with positive probability,

$$p_t(i_t, m_t) = \frac{\mu \sum_{m_S \in M} i_t(m_S|\theta = G) c_t(m_t \mid m_S)}{\sum_{\theta \in \{G,B\}} \Pr(\theta) \sum_{m_S \in M} i_t(m_S|\theta) c_t(m_t \mid m_S)}. \tag{4}$$

and, if $m_t = \{m'_S\} \notin A(m_S)$ for all $m_S \neq m'_S$, then[6]

$$p_t(i_t, \{m_S\}) = \frac{\mu \, i_t(m_S|\theta = G)}{\sum_{\theta \in \{G,B\}} \Pr(\theta) i_t(m_S|\theta)}. \tag{5}$$

3. For each target $t \in \mathcal{R}$, Sender's choice of information structure i_t and communication strategy c_t are optimal given the posterior beliefs of the target, p_t, and of non-targeted receivers, $\boldsymbol{p}_{-t}$:

$$i_t \in \arg\max_{i \in I} \sum_{\theta \in \{G,B\}} \Pr(\theta) \sum_{m_S \in M} i(m_S \mid \theta) \sum_{m_t \in A(m_S)} c_t(m_t \mid m_S) V_t(m_t, p_t, \boldsymbol{p}_{-t}) \tag{6}$$

and, for each observed message $m_S \in \text{supp}(i_t(\cdot \mid \theta))$ and each allowable message $m_t \in A(m_S)$,

$$c_t(m_t \mid m_S) > 0 \Rightarrow m_t \in \arg\max_{m \in A(m_S)} V_t(m, p_t, \boldsymbol{p}_{-t}). \tag{7}$$

[5]Equation (3) is a version of the "no signaling what you don't know" condition of Fudenberg and Tirole (1991).

[6]This last requirement says that, if the communication protocol allows Sender to credibly communicate the message she observes, then the target's belief upon observing such credible communication must coincide with Sender's belief. This is also a version of the "no signaling what you don't know condition".

4. Sender's choice of target is sequentially optimal:

$$t \in \arg\max_{t' \in \mathcal{R}} \sum_{\theta \in \{G,B\}} \Pr(\theta) \sum_{m_S \in M} i_{t'}(m_S \mid \theta) \sum_{m_{t'} \in A(m_S)} c_{t'}(m_{t'} \mid m_S) V_{t'}(m_{t'}, p_{t'}, \boldsymbol{p}_{-t'}). \quad (8)$$

All proofs are in Appendix A.

3 How to persuade a target

We now study how Sender optimally chooses to communicate with the target. To do so, we fix the identity of the target t, and study how receivers' equilibrium behavior affects Sender's private communication with t.

We begin by studying how the target's example—his choice to buy—affects other receivers' beliefs regarding the quality of the widget, and therefore their choice of whether to buy it. Lemma 1 says that a positive example—the target buys—induces other receivers to hold more positive beliefs about the quality of the widget.

Lemma 1 (The power of examples)**.** *In any equilibrium* $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*, and for each receiver* $t \in \mathcal{R}$*, if Sender targets* t*, then receiver* r*'s posterior belief is greater when he observes that* t *buys the widget than when he observes that* t *does not: for all* $r \neq t$*,* $p_r^*(t \mid 1) \geq p_r^*(t \mid 0)$*.*

Lemma 1 says that the target's example has the power to influence other receivers. Intuitively, if other receivers observe that the target t buys the widget, then they infer that the target's posterior belief, p_t, is at least equal to his skepticism, σ_t. Conversely, if they observe that the target does not buy, then they infer that his posterior belief is less than his skepticism. Since the target has access to more information through his communication with Sender, other receivers who observe his example also make inference about the quality of the widget. They infer that the widget is more likely to be good if they observe that the target buys the widget than if they observe that the target does not buy it.

Lemma 1 immediately allows us to establish the following key step towards our main theorem. Lemma 2 says that, on average, a positive example weakly increases the *probability* that non-targeted receivers buy the widget. That is, a positive example may compel other receivers to buy.

Lemma 2 (Examples compel)**.** *In any equilibrium* $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*, and for each receiver* $t \in \mathcal{R}$*, if Sender targets* t*, then receiver* r*'s probability of buying the widget is greater* *if the target buys the widget: for all $r \neq t$,*

$$\pi_t \mathbb{1}[p_r^*(t \mid 1) \geq \sigma_r] + (1 - \pi_t)\mathbb{1}[p_r^*(t \mid \emptyset) \geq \sigma_r] \geq \pi_t \mathbb{1}[p_r^*(t \mid 0) \geq \sigma_r] + (1 - \pi_t)\mathbb{1}[p_r^*(t \mid \emptyset) \geq \sigma_r]. \tag{9}$$

We can now establish our main result regarding how Sender should persuade the target. Theorem 1 says that, in any equilibrium, Sender optimally communicates with the target as if the target was the only receiver: Sender simply maximizes the probability that the target buys the widget. Notice that this feature of equilibrium play holds both on and off the equilibrium path (i.e., in any proper subgame of the targeted persuasion game resulting from Sender choosing a target).

Theorem 1 (Example-unraveling). *In any equilibrium $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$, and for each receiver $t \in \mathcal{R}$, if Sender targets t, then she maximizes the probability that t buys the widget:*

$$i_t^* \in \arg\max_{i\in I} \sum_{\theta\in\{G,B\}} \Pr(\theta) \sum_{m_S\in M} i(m_S \mid \theta) \sum_{m_t\in A(m_S)} c_t^*(m_t \mid m_S)\mathbb{1}[p_t^*(i, m_t) \geq \sigma_t] \tag{10}$$

and, for each observed message $m_S \in \operatorname{supp}(i_t^*(\cdot \mid \theta))$ and each allowable message $m_t \in A(m_S)$,

$$c_t^*(m_t \mid m_S) > 0 \Rightarrow m_t \in \arg\max_{m\in A(m_S)} \mathbb{1}[p_t^*(i_t^*, m) \geq \sigma_t]. \tag{11}$$

Theorem 1 says that, in equilibrium, Sender communicates with any target as if other receivers did not exist, even though Sender's objective is to maximize how many receivers buy the widget and she knows that, with probability π_t, each non-targeted receiver will observe the target's example. This result holds precisely because of Lemma 2: since the probability a non-targeted receiver buys the widget is greater if the target buys the widget, a communication strategy for Sender which maximizes the equilibrium probability that the target buys the widget also maximizes the probability that each other receiver buys it.

The key implication of Theorem 1 is that Sender has limited scope for choosing a target to set an example for other receivers. Ideally, Sender would prefer to commit to a communication strategy that maximizes how many receivers buy the widget—*not* the equilibrium probability the target buys. But other receivers would correctly anticipate that, in the privacy of the Sender-target communication, Sender would renege such a commitment and communicate such as to maximize the probability the target sets a positive example. Therefore, any Sender's plan to fool other receivers will "unravel".[7] In the following section, we use Theorem 1 to study Sender's optimal choice of target.

4 Who to target

We now turn to the question of who Sender should target. Our main observation is that Sender's key tradeoffs are the same across communication protocols.

We first note that the set of possible equilibria can be divided according to whether the target's example has the power to influence other receivers—i.e., whether his example is informative. Proposition 1 says that, independent of the communication protocol, all equilibria are of one of two types. In one, Sender chooses a target and communicates with him in such a way that, in equilibrium, the target buys the widget if and only if he is a fan—i.e., he would have bought the widget in the absence of any further information. Therefore, the target's example provides no useful information to other receivers, and each receiver buys the widget if and only if he is a fan. In contrast, in the other type of equilibrium, Sender optimally chooses a target and communicates with him in such a way that, in equilibrium, the target buys the widget with probability strictly between 0 and 1. In turn, this means that the target's example is informative for other receivers—i.e., in equilibrium, the target's example has (strict) power.

Proposition 1 (Only fans and informative example equilibria)**.** *In any equilibrium* $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*, either:*

1. ***(Only fans equilibrium.)*** *Each receiver* r*, including the target, buys the widget if and only if he is a fan* ($\sigma_r \leq \mu$)*, or*

2. ***(Informative example equilibrium.)*** *Sender's target,* t^**, buys the widget with probability strictly between* 0 *and* 1 *and, for every receiver* $r \neq t^*$*, his posterior belief when he observes that the target buys (respectively, does not buy) the widget is strictly greater (respectively, smaller) than the prior:* $p_r^*(t^* \mid 1) > \mu > p_r^*(t^* \mid 0)$.

We now characterize Sender's equilibrium value of choosing a specific target. To do so, we first recall that, by Theorem 1, in equilibrium Sender communicates with a target—i.e., she chooses i_t^* and c_t^*, given the target's posterior belief p_t^*—so as to maximize the

[7] Another—perhaps subtle—detail of Theorem 1, is that because Sender's equilibrium communication strategy maximizes the *equilibrium* probability that the target buys, it crucially depends on the target's equilibrium belief as a function of the messages that Sender communicates to him. This detail will be important when the communication protocol allows for multiple equilibrium beliefs for the target (see Section 6.2).

target's probability of buying the widget. For a given choice of i_t^*, the messages that may be observed by Sender, $M(i_t^*) \equiv \{m_S \in M : \exists \theta \in \{G, B\},\ i_t^*(m_S \mid \theta) > 0\}$, are divided into two sets: those that allow Sender to communicate a message m_t that induces the target to buy ($p_t^*(i_t^*, m_t) \geq \sigma_t$) and those not allowing such a message. Theorem 1 says that whenever Sender observes a message of the first type, she always prefers to induce the target to buy. Whenever she observes a message of the second type, she cannot do anything else than induce the target not to buy. Therefore, Sender-target equilibrium private communication is completely characterized by an information structure i_t^* and a set $Y_t^*(i_t^*, p_t^*) \subseteq M(i_t^*)$ of observable messages for Sender that, in equilibrium, result in the target buying the widget.

Lemma 3 (Characterizing Sender-target communication)**.** *In any equilibrium* $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*, and for each receiver* $t \in \mathcal{R}$*, there exists a set of messages* $Y_t^*(i_t^*, p_t^*) \subseteq M(i_t^*)$ *such that, if Sender targets* t*, then* t *buys the widget if and only if Sender observes a message in* $Y_t^*(i_t^*, p_t^*)$*.*

In an informative example equilibrium, $Y_{t^*}^* \neq \emptyset$ and $Y_{t^*}^* \subsetneq M(i_{t^*}^*)$, so that a non-targeted receiver $r \neq t^*$ who observes that the target buys the widget will have posterior belief

$$p_r^*(t^* \mid 1) = \Pr(\theta = G \mid m_S \in Y_t^*(i_{t^*}^*, p_{t^*}^*), i_{t^*}^*) > \mu; \tag{12}$$

while one who observes that the target does not buy will have posterior belief

$$p_r^*(t^* \mid 0) = \Pr(\theta = G \mid m_S \notin Y_t^*(i_{t^*}^*, p_{t^*}^*), i_{t^*}^*) < \mu. \tag{13}$$

In contrast, in an only fans equilibrium, $Y_{t^*}^* \in \{\emptyset, M(i_{t^*}^*)\}$, so that non-targeted receivers' posterior always equals the prior, μ.

Sender's value of choosing a target is given by the expected number of receivers that will buy the widget if Sender chooses him. Because powerful examples can compel receivers to buy and not to buy (Lemma 2), Sender is optimally choosing a risky lottery. If the target buys the widget, then his positive example may compel even a skeptical receiver to buy, adding to Sender's sales. Therefore, intuitively, a more popular target is valuable to Sender because his example is bound to compel more receivers. However, such a lottery also carries risks: because Senders has fans, a negative example may compel some of them *not* to buy, reducing Sender's sales. Therefore, a more popular target also carries greater risks.

To formally analyze this tradeoff and study how the equilibrium value of targeting a specific receiver depends on his popularity, it is useful to define the following equilibrium quantities. First, given an information structure i_t^* and a set of messages $Y_t^* \subseteq M(i_t^*)$, let

$$G_t^*(i_t^*, p_t^*) \equiv |\{r \in \mathcal{R} \setminus \{t\} : \mu < \sigma_r \leq \Pr(\theta = G \mid m_S \in Y_t^*(i_t^*, p_t^*), i_t^*)\}| \geq 0 \tag{14}$$

denote the number of skeptical receivers (not equal to t) who, in (any continuation-game) equilibrium, are compelled to buy the widget if they observe t's positive example. This captures the potential *gains* from the target's example. Similarly, let

$$L_t^*(i_t^*, p_t^*) \equiv |\{r \in \mathcal{R} \setminus \{t\} : \Pr(\theta = G \mid m_S \notin Y_t^*(i_t^*, p_t^*), i_t^*) < \sigma_r \leq \mu\}| \geq 0 \tag{15}$$

be the number of fans (not equal to t) who, in (any continuation-game) equilibrium, are compelled not to buy the widget if they observe t's negative example. This captures the potential *losses* from the target's example. Finally, let

$$F_t \equiv \max\{r \in \mathcal{R} \setminus \{t\} : \sigma_r \leq \mu\} \in \{F-1, F\}$$

be the number of fans (not equal to t).

Notice that, because of Theorem 1, the equilibrium probability of Sender observing a message in Y_t^* under i_t^*,

$$P_t^*(i_t^*, p_t^*) \equiv \Pr(m_S \in Y_t^*(i_t^*, p_t^*), i_t^*)$$

is independent of the target's popularity π_t. It follows that $G_t^*(i_t^*, p_t^*)$ and $L_t^*(i_t^*, p_t^*)$ are also independent of the target's popularity. Therefore, we can characterize the equilibrium value of targeting receiver t as a linear function of his popularity, π_t.

Lemma 4 (The equilibrium value of a targeting a receiver)**.** *In any equilibrium* (t^*, $\{i_t^*\}_{t\in\mathcal{R}}$, $\{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}}$)*, and for each receiver $t \in \mathcal{R}$, Sender's value $\mathbb{E}[V_t^*(i_t^*, p_t^*)]$ of targeting t is given by:*

$$\mathbb{E}[V_t^*(i_t^*, p_t^*)] = P_t^*(i_t^*, p_t^*)(1 + \pi_t G_t^*(i_t^*, p_t^*)) - (1 - P_t^*(i_t^*, p_t^*))\pi_t L_t^*(i_t^*, p_t^*) + F_t. \tag{16}$$

An immediate implication of (16) is that the popularity π_t of a target t is a double-edged sword for Sender—whether Sender benefits from a more popular target depends on the potential gains and losses from his example.

Proposition 2 (The double-edged sword of popularity)**.** *In any equilibrium,* (t^*, $\{i_t^*\}_{t\in\mathcal{R}}$, $\{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}}$)*, Sender's value $\mathbb{E}[V_t^*(i_t^*, p_t^*)]$ of targeting receiver $t \in \mathcal{R}$ is:*

1. *increasing with his popularity if the expected gains from his example are greater than the*

expected losses from it: $P_t^*(i_t^*, p_t^*)G_t^*(i_t^*, p_t^*) > (1 - P_t^*(i_t^*, p_t^*))L_t^*(i_t^*, p_t^*)$*,*

2. *decreasing with his popularity if the expected gains from his example are less than the expected losses from it:* $P_t^*(i_t^*, p_t^*)\ G_t^*(i_t^*, p_t^*) < (1 - P_t^*(i_t^*, p_t^*))L_t^*(i_t^*, p_t^*)$*.*

The relative importance of gains and losses from a target's example crucially depend on (i) the probability $P_t^*(i_t^*, p_t^*)$ with which the target buys the widget under Sender's optimal communication and (ii) the distribution of skepticism among other receivers. Therefore, the Example-unraveling theorem and The double-edged sword of popularity jointly drive the choice of the optimal target. In fact, we will show that, in equilibrium, Sender may optimally choose a target "despite" his popularity—i.e., an increase in the target's popularity may induce Sender to switch to a less popular target (see Corollary 1).

In Section 6 we fully characterize equilibrium behavior in two canonical special cases of our model: when communication takes the form of Bayesian persuasion and when it takes the form of information disclosure.[8] The full characterization of equilibrium behavior allows us to discuss (i) Sender's optimal choice of target, (ii) when the target's popularity is, in equilibrium, a positive or a negative feature from Sender's perspective, and (iii) why Sender may benefit from communication protocols that allow her *less* flexibility. Before that, we pause to discuss how our results extend if we relax some of our benchmark assumptions.

5 Discussion of model assumptions

We now discuss how our results change (or do not change) if we relax several of the assumptions made in our benchmark model. We discuss each assumption in a separate subsection. The reader may skip this section without any loss of understanding of subsequent sections.

5.1 Nonlinear Sender's payoff and strategic complementarities

In Online Appendix B, we extend the benchmark model to allow for Sender's payoff to be strictly increasing in receivers' actions and for strategic complementarities between receivers' actions (a receiver's incentive to buy the widget is higher if more other receivers

[8]Another canonical example one might consider is *cheap talk* (Crawford and Sobel, 1982). In our setting cheap talk turns out to be trivial: the only equilibrium is a babbling one.

buy the widget).[9] The former accommodates, for example, when Sender aims to persuade receivers to vote for a bill and she expects each receiver to vote sincerely but abstain with positive probability[10]. The latter accommodates consumption complementarities between receivers.

The Example-unraveling Theorem extends verbatim. Intuitively, observing that the target buys increases both a non-targeted receiver's posterior that the widget is good and his incentive to buy, both because he knows that the target himself has bought the widget and because he expects other non-targets to buy with greater probability. Combined, these mean that the equilibrium distribution over non-targets' actions, conditional on the target buying, first-order stochastically dominates that when the target does not buy. Hence Sender's payoff is greater when the target buys, so that, in equilibrium, she maximizes the probability that the target buys given the strategies and beliefs of non-targets.

Likewise, popularity continues to be a double-edged sword. However, both edges are "sharper" when receivers' actions are strategic complements. In this case, a higher popularity also raises a non-target's belief about whether other receivers buy (not buy) upon observing the target buy (not buy). This means that the marginal effect of an increase in popularity on the number of receivers who buy (do not buy) when the target buys (does not buy) is greater.

5.2 Multiple targets

The benchmark model assumes that Sender can only privately communicate with a single target. In Online Appendix C, we show that the Example-unraveling Theorem extends to a variation of the model in which: (i) Sender chooses a subset of up to $T \in \{1, \ldots, R-1\}$ targets and then privately communicates with each one of them; (ii) all targets then individually and simultaneously choose whether to buy the widget; and (iii) before choosing whether to buy the widget, each non-targeted receiver independently observes at most one targeted receiver's choice with probability determined by the target's popularity. In particular, the Example-unraveling Theorem now implies that Sender maximizes the probability that *each* target buys holding fixed the strategies and beliefs of non-targeted receivers. We further show that even if assumption (ii) is relaxed to allow for Sender's communication to be correlated across targeted receivers, a weaker version

[9]Because of strategic complementarities, for a given set of non-targets' beliefs about the quality of the widget upon observing the target's actions, there can be multiple equilibria in the continuation game between non-targets. We adopt the standard approach in the literature, and assume that receivers play the largest equilibrium, which exists (see Milgrom and Roberts, 1990) and is also the Sender-preferred one.

[10]This is commonly assumed in probabilistic voting models (see Lindbeck and Weibull, 1987).

of the Example-unraveling Theorem continues to hold. Namely, for each targeted receiver, Sender's equilibrium information structure maximizes the probability the target buys among all information structures which induce the same distribution over messages sent to other targets.

5.3 Unobserved target identity

In our benchmark model, Sender's choice of target is public, in the sense that all receivers observe who is targeted, even when they do not observe the target's example. In many applications, it is reasonable to think that Sender's choice of target—and not only her communication with him—is private information, so that other receivers only learn the identity of the target if they observe the target's example.[11] Our analysis can easily accommodate such a variation of our model. In particular, we note that, in any equilibrium, all non-targeted receivers must correctly conjecture Sender's equilibrium choice of target. Therefore, our results, including the Example-unraveling Theorem and The double-edged sword of popularity, hold verbatim.

5.4 Receivers observing other receivers' actions

Our benchmark model makes the simplifying assumption that only the target can set an example. In more complex networks, some non-targeted receivers may be influential enough that their choice of whether to buy the widget is also observed by—and influence the choice of—some other receivers. In Online Appendix D, we extend our results to a simple variation of our model that captures this idea. We assume that, after nature has chosen which non-targeted receiver observes the choice of the target, all receivers make a truthful announcement of their intention to buy, given their (interim) belief about the quality of the widget. For each ordered pair of receivers (r, j), with $r \neq j$, let $o_r(j) \in [0, 1]$ be the probability that r observes j's announcements.[12] Finally, all non-targeted receivers make their choice.[13]

The Example-unraveling Theorem continues to hold in this extension, but examples are more powerful. Intuitively, persuading the target to buy increases the probability that those who observe his example also announce their intention to buy. Hence, the target's

[11] The arguments in this section hold also if non-targeted receivers can only observe positive examples—i.e., if they observe nothing unless the target in fact buys the widget.

[12] A natural special case has $o_r(j) = \pi_j$, so that j's popularity equals the probability that other receivers learn his intentions to buy or not.

[13] Our benchmark model obtains if $o_r(j) = 0$ for all r and j.

positive example spreads to more non-targeted receivers through the influence of those compelled by his example.

Similarly, popularity remains a double-edged sword. However, like when receivers' actions are strategic complements, both edges are "sharper" when the target's example can spread through the announcement of other influential receivers.

5.5 Informed Sender and signaling through targeting

We can also study a variation of our model in which Sender is informed about the quality of the widget, so that θ can also be interpreted as Sender's type, with $\theta = 1$ (respectively, $\theta = 0$) meaning that Sender is good (respectively, bad). We assume that Sender-target communication is sufficiently rich in the sense that good Sender has access to a message that is unavailable to bad Sender.[14] It follows that, in any equilibrium, the target fully learns θ. Therefore, in equilibrium, every non-targeted receiver who observes the target's example also fully learns θ. These two observations guarantee that an analogous result to the Example-unraveling Theorem holds: for a given target, both good and bad Sender maximize the equilibrium probability that the target buys the widget. Furthermore, it is easy to see that the equilibrium expected sales generated by a target depend only on his popularity and *not* on his skepticism. In particular, the double-edged sword of popularity now takes a different form: a target's greater popularity is beneficial to good Sender and damaging for bad Sender. Nevertheless, because Sender is now informed, her choice of target may reveal information about the quality of the widget. The consequence of this is that, in any equilibrium, bad Sender mimics good Sender, lest she reveal her type. I.e., in equilibrium both types of Sender choose the same target. Because a greater target's popularity benefits good Sender, in equilibrium, both good and bad Sender choose to target the receiver with greatest popularity.

6 Optimal targeted persuasion under different communication protocols

6.1 Bayesian Persuasion

We now study optimal targeted persuasion when Sender-target communication takes the form of Bayesian persuasion à la Kamenica and Gentzkow (2011). In this setting, Sender

[14] I.e., there exist a message m_G observed by Sender on which she learns $\theta = G$ and a message m_B on which she learns $\theta = B$ such that $A(m_G) \setminus A(m_B) \neq \emptyset$.

can choose any information structure, but must truthfully communicate to the target the message she observes. I.e., $I = \{i : \Theta \to \Delta M\}$ and $A(m_S) = \{m_S\}$ for all $m_S \in M$. We assume that Sender has access to at least two messages: $|M| > 1$.

We begin by characterizing the value of a target for Sender. Lemma 5 says that, in any equilibrium, the probability that the target buys depends on the target's skepticism, σ_t.

Lemma 5 (The optimal Bayesian persuasion of a target.)**.** *Suppose that communication takes the form of Bayesian persuasion. In any equilibrium* $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*, and for each receiver* $t \in \mathcal{R}$*, if Sender targets* t*, then* t *buys the widget with probability* $\min\{1, \mu/\sigma_t\}$*.*

Intuitively, because under Bayesian persuasion Sender must truthfully reveal her message, for a given information structure, the probability that the target buys the widget coincides with the probability that Sender's own posterior exceeds the target's threshold, σ_t. Then, by Theorem 1, in equilibrium Sender chooses the information structure which maximizes the probability her own posterior exceeds σ_t. If t is a skeptic, this involves choosing an information structure that induces only posteriors $p_t = 0$ (t is sure that the widget is bad) and $p_t = \sigma_t$ (the target is just indifferent between buying and not buying the widget). Bayes plausibility then yields that the probability of inducing $p_t = \sigma_t$ equals μ/σ_t. If instead t is a fan ($\sigma_t \leq \mu$), so that he buys when completely uninformed about the quality of the widget, then this involves choosing any information structure that always induces posteriors under which the target buys.

Lemma 6 characterizes the power of the target's example over non-targeted receivers' behavior. It says that a fan's example has no power, while a skeptic's positive example persuades all receivers who are less skeptical than he is.

Lemma 6 (The power of examples under Bayesian persuasion)**.** *Suppose that communication takes the form of Bayesian persuasion. In any equilibrium* $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*, and for each receiver* $t \in \mathcal{R}$*, if Sender targets* t *then, for all receivers* $r \neq t$*:*

1. ***(If the target is a fan.)*** *If* $\sigma_t \leq \mu$*, then* r *buys the widget if and only if he is a fan* ($\sigma_r \leq \mu$)*.*

2. ***(If the target is a skeptic.)*** *If* $\sigma_t > \mu$*, then* r *buys the widget if and only if either he observes that* t *buys the widget and he is less skeptical than* t ($\sigma_r \leq \sigma_t$) *or he does not observe* t*'s choice and he is a fan* ($\sigma_r \leq \mu$)*.*

Intuitively, when Sender targets a fan, all receivers correctly anticipate that the target will always buy the widget. Therefore, the target's example provides no useful information to other receivers. In contrast, when Sender targets a skeptic, all other receivers correctly anticipate that Sender will optimally persuade the target. Therefore, if they observe that the target buys the widget, they will correctly infer that the information provided to the target is such that the widget is good with probability exactly σ_t. Hence, Receiver r only buys if he is less skeptical than the target: $\sigma_r \leq \sigma_t$. Instead, if non-targeted receivers observe that the target does not buy the widget, they infer that the information provided to the target is such that the widget is surely bad. Hence, they do not buy.

It is useful to recall from Lemma 4 that the value of targeting receiver r depends on the potential gains $G_r^*(i_r^*, p_r^*)$ and losses $L_r^*(i_r^*, p_r^*)$ from his example. In the case of Bayesian Persuasion of a skeptical target, $P_r^*(i_r^*, p_r^*) = \mu/\sigma_r$, $G_r^*(i_r^*, p_r^*) = r - 1$, and $L_r^*(i_r^*, p_r^*) = F$. We can therefore compute the value of choosing a target as follows.

Lemma 7 (The value of a target under Bayesian persuasion)**.** *Suppose that communication takes the form of Bayesian persuasion. In any equilibrium,* $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*, Sender's value* $\mathbb{E}[V_t^*]$ *of targeting receiver* $t \in \mathcal{R}$ *is equal to*

1. ***(If the target is a fan.)*** *the number of fans,* F*, if* $\sigma_t \leq \mu$*;*

2. ***(If the target is a skeptic.)***

$$\underbrace{\mu/\sigma_r}_{\textit{Sale to } t} + \underbrace{\pi_t(t-1)\mu/\sigma_t}_{\textit{Power of } t\textit{'s example}} + \underbrace{(1-\pi_t)F}_{\textit{Sales to fans.}}$$

if $\sigma_t > \mu$*.*

We can now characterize the set of equilibria. Proposition 3 says that if all skeptical receivers are excessively skeptical, then Sender optimally chooses to target a fan. The outcome is equivalent to Sender targeting nobody and only selling the widget to fans. Otherwise, Sender targets the skeptical receiver with the greatest value.

Proposition 3 (Optimal targeted Bayesian persuasion)**.** *Suppose that communication takes the form of Bayesian persuasion. In any equilibrium,* $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*:*

1. ***(Only fans equilibrium.)*** *If for all skeptical receivers* r*,*

$$\sigma_r > \mu \frac{1 + \pi_r(r-1)}{\pi_r F}, \tag{17}$$

then Sender targets a fan ($\sigma_t^* \leq \mu$) *and, along the equilibrium path, each receiver* r *buys if and only if he is a fan* ($\sigma_r \leq \mu$)*.*

2. ***(Bayesian persuasion of a skeptic.)*** *If there exists a skeptical receiver* r *such that*

$$\sigma_r < \mu \frac{1 + \pi_r(r-1)}{\pi_r F}, \tag{18}$$

then Sender targets a skeptic $t^* \in \arg\max_{t\in\mathcal{R}}\{\mu/\sigma_t[1+\pi_t(t-1)]+(1-\pi_t)F\}$ and, along the equilibrium path, each non-targeted receiver r buys if and only if either he observes that t^* buys the widget and he is less skeptical than t^* ($\sigma_r \leq \sigma_{t^*}$) or he does not observe t^*'s choice and he is a fan ($\sigma_r \leq \mu$).

Intuitively, Sender avoids receivers who are too skeptical because they are too hard to persuade. Were they the only receivers, Sender would attempt to persuade them as such an attempt would not cost Sender any sale. But because Senders has fans, attempting to persuade skeptical receivers carries the cost of losing some sales to fans if the fans observe the negative example of skeptical targets. If skeptical receivers are very skeptical, then this cost is very likely to materialize as Sender is unlikely to persuade them. Therefore, Sender may prefer to avoid targeting skeptical receivers altogether.

What makes a skeptical receiver "not too skeptical" for Sender also depends on the receiver's popularity. As we discussed in Proposition 2, greater popularity is a double-edged sword for Sender and may increase (or decrease) the value of targeting a specific receiver depending on the equilibrium expected gains and losses from his example. Corollary 1 precisely pins down, for the case of Bayesian Persuasion, when the expected gains from a target's example outweigh the expected losses from it.

Corollary 1 (The double-edged sword of popularity under Bayesian persuasion)**.** *Suppose that communication takes the form of Bayesian persuasion. In any equilibrium, Sender's value $\mathbb{E}[V_t^*]$ of targeting receiver $t \in \mathcal{R}$ increases with t's popularity, π_t, if $\sigma_t < (t-1)\mu/F$, and decreases with his popularity if $\sigma_t > (t-1)\mu/F$.*

Because (18) can hold for $\sigma_t > (t-1)\mu/F$, Corollary 1 and Proposition 3 imply that a marginal increase in the popularity of the *optimal* target may harm Sender. In fact, when these conditions hold together, an increase in the target's popularity may induce Sender to switch to a less popular target or to a fan.

Intuitively, an increase in the target's popularity entails a trade-off for Sender. On the one hand, a more popular target raises the probability that, if he is persuaded to buy, less skeptical (but not fans) receivers are persuaded to buy by his example. On the other hand, a more popular target also raises the probability that, if he is not persuaded to buy, fans—who would have bought the widget otherwise—are persuaded *not* to buy from observing his example. The first effect arises with probability μ/σ_t and affects at most $t-1-F$ skeptic receivers. The second effect arises with probability $1-\mu/\sigma_t$ and affects F receivers. Hence, the first effect dominates when the target is less skeptical, when there are more skeptical receivers less skeptical than he is, or when there are fewer fans.

6.2 Information disclosure

We now study optimal targeted persuasion when Sender-target communication takes the form of (generalized) information disclosure à la Milgrom (1981). In this setting, Sender is endowed with a single information structure and can only choose what to (truthfully) disclose about the message she observes. That is, $I = \{i\}$, and, for all $m_S \in M$, $A(m_S) \equiv \{m_t \in 2^M : m_S \in m_t\}$. Notice that Sender does not have to communicate all of the information received, i.e., she can communicate $m_t \neq \{m_S\}$. However, she cannot lie about what she observes, i.e., she must choose m_t such that $m_S \in m_t$. We assume that $i(\cdot|\theta)$ has full support.[15] Furthermore, for ease of notation, we let $M \subseteq \mathbb{N}$ and associate smaller messages to smaller beliefs about the widget being good: $m_S < m'_S \Rightarrow \Pr(\theta = 1 \mid m_S) < \Pr(\theta = 1 \mid m'_S)$. As is common in this literature, we focus on the Sender-preferred equilibrium.

As with Bayesian persuasion, we first characterize the value of a target for Sender. Recall by Lemma 3 that, in any equilibrium, the messages Sender observe are partitioned so that there exists a set of observable message such that: when Sender observes a message in this set, her optimal communication induces the target to buy; when Sender observes a message outside of this set, her optimal communication induces the target not to buy. Lemma 8 precisely characterizes the set of possible equilibrium message partitions under information disclosure.

Lemma 8 (The optimal information disclosure to a target)**.** *Suppose that communication takes the form of information disclosure and Sender chooses target $t \in \mathcal{R}$. Let*

$$\mathcal{Y}(t) \equiv \left\{ Y \subseteq M : \begin{array}{l} \frac{\mu i(m_S|G)}{\sum_\theta \Pr(\theta) i(m_S|\theta)} \geq \sigma_t \Rightarrow m_S \in Y, \text{ and} \\ \frac{\mu \sum_{m_S \in Y} i(m_S|G)}{\sum_{m_S \in Y} \sum_\theta \Pr(\theta) i(m_S|\theta)} \geq \sigma_t \end{array} \right\}$$

be the set of subsets Y of messages for Sender (i) containing all messages which, if fully revealed to the target, would induce him to buy, and (ii) that would induce the target to buy were he to know only that Sender has observed a message in Y. Take any pair (p^*, c^*) satisfying (4) and (5). Then, there exists an equilibrium, $(t^*, \{i^*_t\}_{t \in \mathcal{R}}, \{c^*_t\}_{t \in \mathcal{R}}, \{p^*_t, \boldsymbol{p}^*_{-t}\}_{t \in \mathcal{R}})$, with $c^*_t = c^*$ and $p^*_t = p^*$ if and only if there exists $Y \in \mathcal{Y}(t)$ such that the target buys the widget if and only if Sender observes $m_S \in Y$: for all $m_S \in Y$ and m_t such that $c^*(m_t \mid m_S) > 0$, $p^*(i, m_t) \geq \sigma_t$; for all $m_S \notin Y$ and m_t such that $c^*(m_t \mid m_S) > 0$, $p^*(i, m_t) < \sigma_t$.

Intuitively, for any target's posterior belief $p_t(i, m_t)$, messages observed by Sender are naturally divided into two sets: those that allow Sender to communicate a message m_t that

[15] That is, for all $m_S \in M$, there exists a $\theta \in \{G, B\}$ in which $i(m_S|\theta) > 0$.

induces the target to buy ($p_t(i, m_t) \geq \sigma_t$) and those not allowing such a message. Theorem 1 says that whenever Sender observes a message of the first type, she always prefers to induce the target to buy. Whenever she observes a message of the second type, she cannot do anything else than induce the target not to buy. Therefore, in any equilibrium, Sender-target private communication is completely characterized by a set Y of observable message for Sender that, in equilibrium, result in the target buying the widget.

Lemma 9 (The power of example under information disclosure)**.** *Suppose that communication takes the form of information disclosure and Sender chooses target $t \in \mathcal{R}$. For each $Y \in \mathcal{Y}(t)$, let*

$$p_Y(1) \equiv \Pr(\theta = G \mid m_S \in Y) = \frac{\mu \sum_{m_S \in Y} i(m_S|G)}{\sum_\theta \Pr(\theta) i(m_S|\theta)}, \text{ and}$$
$$p_Y(0) \equiv \Pr(\theta = G \mid m_S \notin Y) = \frac{\mu \sum_{m_S \notin Y} i(m_S|G)}{\sum_\theta \Pr(\theta) i(m_S|\theta)}.$$

For any Receiver $r \neq t$, in any equilibrium in which, if targeted, t buys the widget if and only if Sender observes $m_S \in Y$, r buys the widget if and only if he either: observes that t buys the widget and $\sigma_r \leq p_Y(1)$; observes that t does not buy the widget and $\sigma_r \leq p_Y(0)$; or he does not observe t's choice and he is a fan ($\sigma_r \leq \mu$).

Intuitively, non-targeted receivers correctly anticipate that, in equilibrium, Sender persuades the target to buy if and only if she observes a message in Y. Hence, upon observing that the target buys the widget, they must conclude that the widget is good with probability $p_Y(1)$—and hence they buy if they are not more skeptical than $p_Y(1)$. Instead, if they observe that he does not buy, they conclude that the widget is good with probability $p_Y(0)$—and hence they buy if they are not more skeptical than $p_Y(0)$.

We can now compute the value of choosing a target.

Lemma 10 (The value of a target under information disclosure)**.** *Suppose that communication takes the form of information disclosure. For each $Y \in \mathcal{Y}(t)$, let $i(Y)$ be the probability that Sender observes a message $m_S \in Y$. Then in any Sender-preferred equilibrium, Sender's value $\mathbb{E}[V_t]$ of choosing target $t \in \mathcal{R}$ equals*

$$\max_{Y \in \mathcal{Y}(t)} \left\{ \begin{array}{c} i(Y) + \pi_t \Big(i(Y) \Big| \{r \neq t : \sigma_r \leq p_Y(1)\} \Big| + \\ (1 - i(Y)) \Big| \{r \neq t : \sigma_r \leq p_Y(0)\} \Big| \Big) + (1 - \pi_t)(F - \mathbb{1}[t \leq F]) \end{array} \right\}. \tag{19}$$

We can now characterize the set of Sender-preferred equilibria. Intuitively, only two types of equilibria may arise. First, Sender may choose a target and communicate with him so that the target either buys or does not buy with certainty ($Y \in \{\emptyset, M\}$). In this case, the target's example bears no information to other receivers and we have an *only fans* equilibrium: a receiver buys if and only if he is a fan.

Second, Sender may choose a target and communicate with him in such a way that the target buys the widget with probability strictly between 0 and 1 ($Y \notin \{\emptyset, M\}$)—i.e., an *informative example* equilibrium. Lemma 11 says that, in any such equilibrium, the target's positive example is strong enough to persuade at least one skeptic to buy. Notably, the target himself may not be a skeptic. That is, unlike in Bayesian Persuasion (see Part 2 of Proposition 3), Sender may now target a receiver with the aim of indirectly persuading receivers that are *more skeptical* than the target.

Lemma 11 (Sender aims to persuade skeptics)**.** *Suppose that communication takes the form of information disclosure. In any informative example Sender-preferred equilibrium in which Sender targets $t^* \in \mathcal{R}$, t^* buys the widget if and only if Sender observes $m_S \in Y \notin \{\emptyset, M\}$. Moreover, there exists a skeptical receiver $r \geq t^*$ that buys the widget upon observing that the target does so: $p_r(t^* \mid 1) = p_Y(1) \geq \sigma_r$.*

Proposition 4 says that if all receivers are excessively skeptical, then Sender optimally chooses to sell only to fans. Otherwise, Sender chooses a target—but not necessarily a skeptical target—and communicates with him in such a way that the target's positive example suffices to compel at least one skeptical receiver to buy the widget.

Proposition 4 (Optimal targeted information disclosure)**.** *Suppose that communication takes the form of information disclosure. In equilibrium:*

1. ***(Only fans equilibrium.)*** *If for all receivers $t \in \mathcal{R}$ and all $Y \in \mathcal{Y}(t) \cap \cup_{r>F}\mathcal{Y}(r)$,*

$$i(Y)|\{r \neq t : \sigma_r \leq p_Y(1)\}| + (1 - i(Y))|\{r \neq t : \sigma_r \leq p_Y(0)\}| > \frac{F - \mathbb{I}[r \leq F] - i(Y)}{\pi_t} \tag{20}$$

   then, along the equilibrium path, each receiver r', including the target, buys if and only if he is a fan ($\sigma_{r'} \leq \mu$).

2. ***(Disclosure to persuade a skeptic.)*** *If there exists a receiver $t \in \mathcal{R}$ and $Y \in \mathcal{Y}(t) \cap \bigcup_{r>F}\mathcal{Y}(r)$ such that*

$$i(Y)|\{r \neq t : \sigma_r \leq p_Y(1)\}| + (1 - i(Y))|\{r \neq t : \sigma_r \leq p_Y(0)\}| > \frac{F - \mathbb{I}[r \leq F] - i(Y)}{\pi_t}, \tag{21}$$

then there exists a skeptic $\overline{r} > F$ and a fan $\underline{r} \leq F$ such that, along the equilibrium path, the target t^* buys if and only if the sender observes $m_S \in Y$, and each non-targeted receiver r' buys if and only if either he observes that t^* buys the widget and he is less skeptical than $\overline{r}$ ($\sigma_{r'} \leq \sigma_{\overline{r}}$), he observes that t^* does not buy the widget and he is less skeptical than $\underline{r}$ ($\sigma_{r'} \leq \sigma_{\underline{r}}$), or he does not observe t^*'s choice and he is a fan ($\sigma_{r'} \leq \mu$).

Corollary 2 characterizes the double-edged sword of popularity (Proposition 2) when Sender-target communication takes the form of information disclosure.

Corollary 2 (The double-edged sword of popularity under information disclosure)**.** *Suppose that communication takes the form of information disclosure. In any equilibrium, Sender's value $\mathbb{E}[V_t^*]$ of targeting receiver $t \in \mathcal{R}$ increases with his popularity π_t if*

$$\min_{Y \in \mathcal{Y}(t)} \left\{ i(Y)|\{r \neq t : \sigma_r \leq p_Y(1)\}| + (1 - i(Y))|\{r \neq t : \sigma_r \leq p_Y(0)\}| \right\} \geq F - \mathbb{1}[t \leq F],$$

and decreases with his popularity if

$$\max_{Y \in \mathcal{Y}(t)} \left\{ i(Y)|\{r \neq t : \sigma_r \leq p_Y(1)\}| + (1 - i(Y))|\{r \neq t : \sigma_r \leq p_Y(0)\}| \right\} \leq F - \mathbb{1}[t \leq F].$$

The main difference between optimal targeted persuasion when communication takes the form of Bayesian persuasion or information disclosure is that, under information disclosure, Sender may choose a target and communicate with him so that the target's positive example compels even more skeptical receivers to buy. In contrast, under Bayesian persuasion, Theorem 1 implies that Sender can never persuade a more skeptical receiver than the target to buy the widget. We now explore the intuition behind this difference and its implications.

6.3 Comparing Bayesian persuasion and information disclosure

We now compare Sender's payoff when communication takes the form of Bayesian Persuasion to when it takes the form of information disclosure. It may seem intuitive that Sender always prefers Bayesian persuasion because this communication protocol affords Sender the flexibility of choosing the information structure, i_t. Thus, Sender has greater freedom in choosing how to persuade the target. However, we show that this is not necessarily the case in our setting.

Under Bayesian persuasion, Sender optimally chooses both the information structure i_t and what to communicates to the target, and *both* are privately observed only by Sender and the target, but not by other receivers. Sender's objective is to persuade as many receivers as possible. Therefore, ideally, Sender would choose a very popular target and commit to an information structure that maximizes the power of his example. However, as discussed in Section 3, the Example-unraveling Theorem establishes that, in equilibrium, such a plan unravels. This is because holding fixed the equilibrium beliefs of non-targeted receivers, Sender strictly prefers to deviate to the information structure that maximizes the *equilibrium* probability that the target buys, so that a positive example will induce to buy only receivers less skeptical than the target and a negative example would induce even fans not to buy (see Lemma 6). Therefore, sometimes Sender may prefer to target a less popular receiver, or even a fan.

In contrast, when communication takes the form of information disclosure, Sender *is* committed to the only information structure available to her. Thus, even though they know Sender will maximize the probability that the target buys, other receivers do not need to conjecture what information structure was chosen by Sender—she must have used the only one available to her. It follows that a target's positive example induces non-targeted receivers to hold beliefs that are, generically, greater than the target's exact skepticism, thus compelling even receivers more skeptical than he is to buy. Therefore, in equilibrium, because Sender is less free to choose how to communicate to the target, Sender is more free to optimally choose a target who is more popular and therefore whose example may persuade more receivers to buy.

Proposition 5 gives sufficient conditions for the existence of an information disclosure structure i such that Sender's expected payoff is greater under information disclosure than under Bayesian persuasion.

Proposition 5 (When Sender strictly prefers information disclosure)**.** *Suppose that under Bayesian persuasion Sender optimally targets a skeptical receiver t. If*

1. $\sigma_t < (t-1)\mu/F$*, so that a marginal increase in t's popularity strictly increases the Sender's value of choosing t, and*

2. *there exists a skeptical Receiver $r < t$ with $\pi_r > \pi_t$, so that r is strictly more popular than t,*

then there exists an information structure i such that Sender's equilibrium expected payoff under information disclosure with $I = \{i\}$ is strictly greater than her equilibrium expected payoff under Bayesian persuasion.

We illustrate this result in an example with three receivers.

Example 1 (Sender strictly prefers information disclosure). *Suppose $F = 1$ and $R = 3$, so there is exactly one fan and two skeptics, $M = \{0, 1\}$, so the message space is binary, and*

$$\pi_2 > \pi_3; \tag{22}$$

$$\frac{\mu}{\sigma_3}(1 + 2\pi_3) + (1 - \pi_3) \geq \max\left\{1, \frac{\mu}{\sigma_2}(1 + \pi_2) + (1 - \pi_2)\right\}; \tag{23}$$

$$\frac{\mu}{\sigma_3} \geq \frac{1}{2}. \tag{24}$$

Notice that equation (22) says that the less skeptical Receiver 2 is more popular than the more skeptical Receiver 3. Equation (23) says that, under Bayesian persuasion, the value of targeting Receiver 3 is greater than the value of targeting Receiver 2 or the value of targeting the only fan. Equation (24) says that (by Corollary 1) a marginal increase in Receiver 3's popularity would increase Sender's value of targeting him.

Finally, suppose that, under information disclosure, Sender is endowed with $i = i^*$ such that $i^*(0|G) = 0$, $i^*(1|G) = 1$, and

$$i^*(1|B) = \frac{\mu(1 - \sigma_2)}{(1 - \mu)\sigma_2}.$$

Bayesian persuasion. *Suppose communication takes the form of Bayesian Persuasion. By Lemma 7 and (23), Sender optimally targets the most skeptical Receiver 3, and her expected payoff is*

$$\frac{\mu}{\sigma_3}(1 + 2\pi_3) + (1 - \pi_3). \tag{25}$$

Notice that in this case Sender optimally chooses $i_3 = i^*$.

Information disclosure. *Suppose communication takes the form of information disclosure. It is straightforward to see that Sender can target Receiver 3, optimally communicate with him under i^*, and expect the same payoff as under Bayesian persuasion. However, we now show that Sender can target the less skeptical—but more popular—Receiver 2, and expect a greater payoff.*

To see this, suppose Sender chooses $t = 2$. By Lemma 8, in the unique (continuation-game) equilibrium the target buys the widget if and only if Sender observes $m_S = 1$. Furthermore, by Lemma 10, Sender's (continuation-equilibrium) expected payoff is given by

$$\frac{\mu}{\sigma_3}(1 + 2\pi_2) + (1 - \pi_2) > \frac{\mu}{\sigma_3}(1 + 2\pi_3) + (1 - \pi_3) \tag{26}$$

where the last inequality follows from (22) and (24).

The key to the example is that, because Sender can credibly commit to use i^*, under information disclosure she can, by targeting Receiver 2, also persuade receiver $r = 3$ if he observes 2's example.[16] In contrast, under Bayesian persuasion, by targeting Receiver 2, Sender is giving up any chance to persuade the more skeptical Receiver 3. The only way to persuade both is to target Receiver 3 and hope that Receiver 2 will observe 3's example. But because Receiver 2 is more popular than Receiver 3, all else equal it is more likely that Receiver 3 will observe 2's example than vice versa. Thus, the extra opportunity of targeting 2 and affect 3 via the example afforded by information disclosure increases Sender's equilibrium payoff. Finally, it is instructive to notice that this result crucially depends on popularity being a positive feature of a target (i.e., it relies on (24)).

7 Conclusions

When choosing who to target in networking activities, salespersons, entrepreneurs, or lobbyists need to anticipate how persuading a target will affect other potential customers, investors, or politicians. Our Example-unraveling theorem allows us to characterize optimal targeted persuasion independently of the specific assumptions about how the communication with a target takes place. An implication of this result captures the intuitive idea that the popularity of a target is a double-edged sword for Sender. We showed how to employ these result to characterize the optimal choice of target under two canonical communication models: Bayesian persuasion and (generalized) information disclosure. As discussed in Section 5, these results extend to a broad class of models, including when Sender is informed before choosing the target or when Sender can choose many targets or when the choice of target is only revealed through his example.[17]

One of our key results is that the popularity of a target is a double-edged sword. Thus, the optimal target may not be the customer or investor with the greatest visibility among other customers or investors. However, when popularity positively affects the value of a target, we showed that Sender may benefit from communication forms that afford her *less* flexibility but allow her to more freely choose who to target based on their popularity. Thus, for example, when hard information is produced independently of her choice, a startup entrepreneur will more likely optimally target a very popular investor—an "influencer" with great visibility among other investors. In contrast, when hard information is produced through experiments and demonstrations designed by her, a startup

[16] By a continuity argument, this holds even for information structures "close" to i^*.

[17] Nevertheless, we note that our result crucially hinges on the assumption that receivers have common values in the sense that all prefer buying a good widget over not buying any widget and prefer both over buying a bad widget. Violating this assumption "inverts" the meaning of the target's example and therefore Sender may not prefer to maximize the probability that the target buys.

entrepreneur will more likely base her targeting choice on the target's skepticism, even at the expense of losing some visibility among other potential investors.

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A Appendix: Proofs

Proof of Lemma 1. Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be an equilibrium and t be any receiver. If t is targeted by Sender, then, by (2), for any receiver $r \neq t$,

$$\begin{aligned}
p_r^*(t \mid 1) &= \frac{\sum_{\substack{m_t\in A(m_S):\\ p_t^*(i_t,m_t)\geq\sigma_t}} \mu \sum_{m_S\in M} i_t^*(m_S|\theta=G)c_t(m_t \mid m_S)}{\sum_{\substack{m_t\in A(m_S):\\ p_t^*(i_t,m_t)\geq\sigma_t}} \sum_{\theta\in\{G,B\}} \Pr(\theta) \sum_{m_S\in M} i_t^*(m_S|\theta)c_t(m_t \mid m_S)} \\
&= \sum_{\substack{m_t\in A(m_S):\\ p_t^*(i_t,m_t)\geq\sigma_t}} \left(p_t^*(i_t, m_t) \times \frac{\sum_{\theta\in\{G,B\}} \Pr(\theta) \sum_{m_S\in M} i_t^*(m_S|\theta)c_t(m_t \mid m_S)}{\sum_{\substack{m_t\in A(m_S):\\ p_t^*(i_t,m_t)\geq\sigma_t}} \sum_{\theta\in\{G,B\}} \Pr(\theta) \sum_{m_S\in M} i_t^*(m_S|\theta)c_t(m_t \mid m_S)} \right) \\
&\geq \sum_{\substack{m_t\in A(m_S):\\ p_t^*(i_t,m_t)\geq\sigma_t}} \left(\sigma_t \times \frac{\sum_{\theta\in\{G,B\}} \Pr(\theta) \sum_{m_S\in M} i_t^*(m_S|\theta)c_t(m_t \mid m_S)}{\sum_{\substack{m_t\in A(m_S):\\ p_t^*(i_t,m_t)\geq\sigma_t}} \sum_{\theta\in\{G,B\}} \Pr(\theta) \sum_{m_S\in M} i_t^*(m_S|\theta)c_t(m_t \mid m_S)} \right) \\
&= \sigma_t,
\end{aligned}$$

where the inequality holds because, conditional on observing message m_t, if t buys, then his posterior belief satisfies $p_t^*(i_t, m_t) \geq \sigma_t$. Similarly, by (2),

$$\begin{aligned}
p_r^*(t \mid 0) &= \frac{\sum_{\substack{m_t\in A(m_S):\\ p_t^*(i_t,m_t)<\sigma_t}} \mu \sum_{m_S\in M} i_t^*(m_S|\theta=G)c_t(m_t \mid m_S)}{\sum_{\substack{m_t\in A(m_S):\\ p_t^*(i_t,m_t)<\sigma_t}} \sum_{\theta\in\{G,B\}} \Pr(\theta) \sum_{m_S\in M} i_t^*(m_S|\theta)c_t(m_t \mid m_S)} \\
&= \sum_{\substack{m_t\in A(m_S):\\ p_t^*(i_t,m_t)<\sigma_t}} \left(p_t^*(i_t, m_t) \times \frac{\sum_{\theta\in\{G,B\}} \Pr(\theta) \sum_{m_S\in M} i_t^*(m_S|\theta)c_t(m_t \mid m_S)}{\sum_{\substack{m_t\in A(m_S):\\ p_t^*(i_t,m_t)<\sigma_t}} \sum_{\theta\in\{G,B\}} \Pr(\theta) \sum_{m_S\in M} i_t^*(m_S|\theta)c_t(m_t \mid m_S)} \right) \\
&\leq \sigma_t
\end{aligned}$$

where the inequality holds because, conditional on observing message m_t, if t does not buy, then his posterior belief satisfies $p_t^*(i_t, m_t) < \sigma_t$. Combining these two results yields $p_r^*(t \mid 1) \geq p_r^*(t \mid 0)$. □

Proof of Lemma 2. Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be an equilibrium and $t \in \mathcal{R}$ be any receiver. Notice that (9) is satisfied if and only if $p_r^*(t \mid 1) \geq p_r^*(t \mid 0)$. By Lemma 1, this is satisfied for all $r \neq t$. □

Proof of Theorem 1. Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be an equilibrium and $t \in \mathcal{R}$ be any receiver. We will show that

$$\arg\max_{m\in A(m_S)} V_t(m, p_t^*, \boldsymbol{p}_{-t}^*) \subseteq \arg\max_{m\in A(m_S)} \mathbb{1}[p_t^*(i_t^*, m) \geq \sigma_t]. \tag{27}$$

If so, by (7), we conclude that (11) holds; by (6), we conclude that (10) holds.

There are two cases to consider. First, suppose that, for all $m_S \in \text{supp}(i_t^*(\cdot|\theta))$ and $m_t \in A(m_S)$, $p_t^*(i_t, m_t) < \sigma_t$. Then every message induces the same action from the target. Hence, the left and right hand sides of (27) coincide.

Second, suppose that there exists at least one $\overline{m}_S \in \text{supp}(i_t^*(\cdot|\theta))$ and $\overline{m}_t \in A(m_S)$ for which $p_t^*(i_t, \overline{m}_t) \geq \sigma_t$. Take any $m_t \in \arg\max_{m\in A(m_S)} V_t(m, p_t^*, \boldsymbol{p}_{-t}^*)$. Then,

$$\begin{aligned}
V_t(m_t, p_t^*, \boldsymbol{p}_{-t}^*) - V_t(\overline{m}_t, p_t^*, \boldsymbol{p}_{-t}^*) = \\
&= (\mathbb{1}[p_t^*(i_t^*, m_t) \geq \sigma_t] - \mathbb{1}[p_t^*(i_t^*, \overline{m}_t) \geq \sigma_t]) \left(1 + \sum_{r\neq t} \pi_t \mathbb{1}[p_r^*(t \mid 1) \geq \sigma_r]\right) \\
&\quad + (\mathbb{1}[p_t^*(i_t^*, m_t) < \sigma_t] - \mathbb{1}[p_t^*(i_t^*, \overline{m}_t) < \sigma_t]) \sum_{r\neq t} \pi_t \mathbb{1}[p_r^*(t \mid 0) \geq \sigma_r] \\
&= (\mathbb{1}[p_t^*(i_t, m_t) \geq \sigma_t] - 1) \left(1 + \sum_{r\neq t} \pi_t \mathbb{1}[p_r^*(t \mid 1) \geq \sigma_r]\right) \\
&\quad + \mathbb{1}[p_t^*(i_t^*, m_t) < \sigma_t] \sum_{r\neq t} \pi_t \mathbb{1}[p_r^*(t \mid 0) \geq \sigma_r] \\
&\geq \mathbb{1}[p_t^*(i_t, m_t) \geq \sigma_t] - 1 \\
&\quad + (\mathbb{1}[p_t^*(i_t, m_t) \geq \sigma_t] + \mathbb{1}[p_t^*(i_t, m_t) < \sigma_t] - 1) \sum_{r\neq t} \pi_t \mathbb{1}[p_r^*(t \mid 0) \geq \sigma_r] \\
&= \mathbb{1}[p_t^*(i_t, m_t) \geq \sigma_t] - 1,
\end{aligned}$$

where the inequality on the second last line holds as, by Lemma 2, $\mathbb{1}[p_r^*(t \mid 1) \geq \sigma_r] \geq \mathbb{1}[p_r^*(t \mid 0) \geq \sigma_r]$. Since the last term must be positive, $p_t^*(i_t, m_t) \geq \sigma_t$ holds. Therefore, $m_t \in \arg\max_{m\in A(m_S)} \mathbb{1}[p_r^*(t \mid 1) \geq \sigma_r]$. □

Proof of Proposition 1. Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be an equilibrium. If t^* buys upon observing message m_t (respectively, does not buy), then $p_{t^*}(i_{t^*}^*, m_{t^*}) \geq \sigma_{t^*}$ (respectively, $p_{t^*}(i_{t^*}^*, m_{t^*}) < \sigma_{t^*}$). Note first that, for any $r \neq t^*$, the posterior belief $p_r^*(t^* \mid 1)$ (respectively, $p_r^*(t^* \mid 0)$) conditional on observing the target buying (respectively, not buying) equals t^*'s average posterior belief across all messages inducing him to buy (respectively, not buy). Therefore, $p_r^*(t^* \mid 1) \geq \mu \geq p_r^*(t^* \mid 0)$.

We divide the proof in two complementary cases. First, suppose that t^* buys with probability 1 or 0. By the law of iterated expectations, the average posterior over all messages equals the prior μ. Therefore, we have that t^* buys if and only if he is a fan. Furthermore, $p_r^*(t^* \mid 1) = p_r^*(t^* \mid 0) = p_r^*(t^* \mid \emptyset) = \mu$, so that receiver r buys if and only if he is a fan. This is an only fans equilibrium. Second, suppose that t^* buys with probability $P \in (0,1)$. Using (1)–(3), we have $p_r^*(t^* \mid 1) > \mu > p_r^*(t^* \mid 0)$. This is an informative example equilibrium. □

Proof of Lemma 3. Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be an equilibrium and $t \in \mathcal{R}$ be any receiver. Let $Y_t^*(i_t^*, p_t^*)$ denote the subset of messages in M drawn with positive probability under i_t^* for which there is an allowable message that persuades the target to buy under belief p_t^*:

$$Y_t^*(i_t^*, p_t^*) \equiv \left\{ m_S \in M : \begin{array}{c} i(m_S|G)\mu + i(m_S|B)(1-\mu) > 0, \text{ and} \\ \exists m_t \in A(m_S) \text{ s.t. } p_t(i_t^*, m_t) \geq \sigma_t \end{array} \right\}.$$

By Theorem 1, for all $m_S \in Y_t^*(i_t^*, p_t^*)$, in equilibrium, when she observes m_S, any message that Sender sends to t with positive probability persuades the target to buy. Thus, the target buys with probability 1 conditional on Sender observing any such m_S. Furthermore, for all $m_S \notin Y_t^*(i_t^*, p_t^*)$, either Sender never observes the message under i_t^*, or Sender observes the message, but there exists no message Sender can send to the target to persuade the target to buy. Hence, the target buys with probability zero conditional on Sender observing any such m_S. □

Proof of Lemma 4. Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be an equilibrium and $t \in \mathcal{R}$ any receiver. If Sender targets t, then conditional on not observing t's action, F_t non-targeted receivers buy the widget. Conditional on observing t's action, if receiver t buys and a non-target receiver $r \neq t$ observes receiver t buy (with probability π_t), receiver r's posterior is $\Pr(\theta = G|m_S \in Y_t^*(i_t^*, p_t^*), i_t^*)$. Hence, the non-target receiver buys if and only if $\Pr(\theta = G|m_S \in Y_t^*(i_t^*, p_t^*), i_t^*) \geq \sigma_r$. An analogous argument implies that conditional on observing the target receiver not buy, a non-targeted receiver r buys if and only if $\Pr(\theta = G|m_S \notin$

$Y_t^*(i_t^*, p_t^*), i_t^*) \geq \sigma_r$. Hence, Sender's value from targeting receiver r is

$$\mathbb{E}[V_t^*(i_t^*, p_t^*)] = P_t^*(i_t^*, p_t^*) \times \begin{pmatrix} 1 + (1-\pi_t) \times F_t \\ +\pi_t \times |\{r \neq t : Pr(\theta = G | m_S \in Y_t^*(i_t^*, p_t^*), i_t^*) \geq \sigma_r\}| \end{pmatrix}$$

$$+ (1 - P_t^*(i_t^*, p_t^*)) \times \begin{pmatrix} 1 + (1-\pi_t) \times F_t \\ +\pi_t \times |\{r \neq t : Pr(\theta = G | m_S \notin Y_t^*(i_t^*, p_t^*), i_t^*) \geq \sigma_r\}| \end{pmatrix}$$

$$= P_t^*(i_t^*, p_t^*)(1 + \pi_t(G_t^*(i_t^*, p_t^*))) - (1 - P_t^*(i_t^*, p_t^*))L_t^*(i_t^*, p_t^*)$$

where the second equality holds as

$$|\{r \neq t : Pr(\theta = G | m_S \in Y_t^*(i_t^*, p_t^*), i_t^*) \geq \sigma_r\}| - F_t = G_t^*(i_t^*, p_t^*)$$
$$F_t - |\{r \neq t : Pr(\theta = G | m_S \notin Y_t^*(i_t^*, p_t^*), i_t^*) \geq \sigma_r\}| = L_t^*(i_t^*, p_t^*) \qquad \square$$

Proof of Proposition 2. Follows immediately from Lemma 4 and noting that, by Theorem 1, the equilibrium quantities $P_t^*(i_t^*, p_t^*)$, $G_t^*(i_t^*, p_t^*)$, $L_t^*(i_t^*, p_t^*)$, and F_t do not depend on π_t. $\square$

Proof of Lemma 5. Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be an equilibrium and $t \in \mathcal{R}$ any receiver. Under Bayesian persuasion, Sender must truthfully communicate whichever message is observed under their chosen information structure $i_t^* \in I$. Hence conditional on Sender observing a message $m_S \in M$, the target's posterior belief coincides with the Sender's posterior belief. This implies that Sender's problem can be reduced to choosing a distribution over posterior beliefs, where the Sender's payoff conditional on posterior belief q is given by $\mathbb{1}[q \geq \sigma_t]$. By Kamenica and Gentzkow (2011), the maximum payoff the Sender can achieve is equal to the concavification of $\mathbb{1}[q \geq \sigma_t]$ evaluated at the prior belief μ, which is $\min\{1, \frac{\mu}{\sigma_t}\}$, and that this can be achieved if Sender can choose an information structure with at least two messages. That the Sender in our setting indeed has access to at least two messages completes our proof. $\square$

Proof of Lemma 6. Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be an equilibrium and $t \in \mathcal{R}$ any receiver. Recall by Lemma 5 that the Sender's payoff is achieved by the concavification of $\mathbb{1}[q \geq \sigma_t]$ evaluated at the prior belief μ. Any information structure i_t^* which achieves this must have the following properties.

1. The set of messages that Sender observes under i_t^* with strictly positive probability can be divided into two: $2^M = M_1 \cup M_0$, where $M_1 \cap M_0 = \emptyset$.

2. For all $m_S \in M_0$, $i_t^*(m_S|G) = 0$, so the target (and Sender) is certain the quality is bad under m_S.

3. For all $m_S \in M_1$, the posterior induced $p_t^*(i_t^*, \{m_S\})$ satisfies $p_t^*(i_t^*, \{m_S\}) \geq \sigma_t$, and

$$\sum_{\theta \in \{G,B\}} \sum_{m_S \in M} p_t^*(i_t^*, \{m_S\}) i^*(m_S|\theta) \Pr(\theta) = \min\{\mu, \sigma_t\},$$

so the target always has a belief of at least σ_t that the quality is good upon observing $m_S \in M_1$, and his average belief coincides with $\min\{\mu, \sigma_t\}$.

By (1) and (2), a non-target receiver r's beliefs that the widget is good conditional on observing that the target buys ($o_t = 1$) and does not buy ($o_t = 0$) are, respectively, $p_r^*(t \mid 1) = \min\{\mu, \sigma_t\}$ and $p_r^*(t \mid 0) = 0$. Parts 1 and 2 of Lemma 6 are then easily verified by comparing $p_r^*(t \mid 1)$ and $p_r^*(t \mid 0)$ and $p_r^*(t \mid \emptyset)$ against r's level of skepticism σ_r. □

Proof of Lemma 7. Let $(t^*, \{i_t^*\}_{t \in \mathcal{R}}, \{c_t^*\}_{t \in \mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t \in \mathcal{R}})$ be an equilibrium and $t \in \mathcal{R}$ any receiver. First, suppose t is a fan. By Lemma 5, in the continuation equilibrium where t is targeted, t buys the widget with probability 1, and by Lemma 6 non-targeted receivers $r \neq t$ with $\sigma_r \leq \mu$ buy the widget. So the value $V_t(m_t, p_t^*, \boldsymbol{p}_{-t}^*)$ to the Sender must be $1 + (F - 1) = F$ for any message m_t sent with positive probability in equilibrium, and it follows that $\mathbb{E}[V_t] = F$.

Now suppose t is a skeptic. By Lemma 5, the target buys the widget with probability μ/σ_t, and, by Lemma 6, each non-targeted receiver $r \neq t$ buys the widget either if they observe that t buys the widget and $\sigma_r \leq \sigma_t$, or if they do not observe that t buys the widget and $\sigma_r \leq \mu$. In particular, $p_r^*(t \mid 1) = \sigma_t$ and $p_r^*(t \mid 0) = 0$. So the expected value, $\mathbb{E}[V_t(m_t, p_t^*, \boldsymbol{p}_{-t}^*)]$, to the Sender is

$$\begin{aligned}
\mathbb{E}[V_t(m_t, p_t^*, \boldsymbol{p}_{-t}^*)] &= \frac{\mu}{\sigma_t}(1 + \pi_t(t - 1 - F) + F) + \left(1 - \frac{\mu}{\sigma_t}\right)(1 - \pi_t)F \\
&= \frac{\mu}{\sigma_t} + \frac{\mu}{\sigma_t}((1 - \pi_t)F + \pi_t(t - 1)) + (1 - \pi_t)F - \frac{\mu}{\sigma_t}(1 - \pi_t)F \\
&= \frac{\mu}{\sigma_t} + \pi_t(t - 1)\frac{\mu}{\sigma_t} + (1 - \pi_t)F,
\end{aligned}$$

as claimed. □

Proof of Proposition 3. By Lemma 7, Sender weakly prefers targeting skeptic r over every fan if and only if $\mu/\sigma_r + \pi_r(r - 1)\mu/\sigma_t + (1 - \pi_r)F \geq F$. Rearranging this yields $\sigma_r \leq \mu(1 + \pi_r(r - 1))/\pi_r F$. Hence: if (17) holds for all $r > F$, then Sender strictly prefers targeting some fan over every skeptic; if (18) holds for at least one $r > F$, then Sender prefers targeting some skeptic over every fan. The remainder of the proposition then follows directly from Lemma 6. $\square$

Proof of Corollary 1. Take any receiver $t \in \mathcal{R}$. If t is a fan, then $V_t = F$, which is constant in π_t. If t is a skeptic, then by Lemma 7, $V_t = \mu/\sigma_t + \pi(t-1)\mu/\sigma_t + (1-\pi_t)F$, which is increasing in π_t if and only if $\sigma_t \leq (t-1)\mu/F$. $\square$

Proof of Lemma 8. We prove necessity and sufficiency separately.

Necessity: Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be any equilibrium with $c_t^* = c^*$ and $p_t^* = p^*$, and $t \in \mathcal{R}$ any receiver. Let $Y \equiv \{m_S : \exists m_T \in A(m_S) \text{ s.t. } c^*(m_T|m_S) > 0 \text{ and } p^*(i, m_t) \geq \sigma_t\}$ denote the set of messages received by the Sender in which, conditional on targeting t, Sender persuades t to buy the widget with positive probability.

First, we claim that t buys the widget if and only if Sender observes $m_S \in Y$. This holds because, by the Example-unraveling Theorem, if the Sender observes some $m_S \in Y$, so there exists a message that Sender could send which persuades the target to buy, then every message sent by the Sender must persuade the target to buy.

Second, we claim that $Y \in \mathcal{Y}(t)$. Because $\{m_S'\} \notin A(m_S)$ for all $m_S' \neq m_S$, we have $p^*(i, \{m_S\}) = \mu i(m_S|G)/(\sum_\theta \Pr(\theta) i(m_S|\theta))$. Hence, if $\mu i(m_S|G)/(\sum_\theta \Pr(\theta) i(m_S|\theta)) \geq \sigma_t$, so fully revealing the message to the target would persuade the target to buy the widget, then $m_S \in Y$. Furthermore, since for all $m_S \in Y$ and $m_T \in A(m_S)$, $c^*(m_T|m_S) > 0$ implies $p^*(i, m_t) \geq \sigma_t$,

$$
\begin{aligned}
&\frac{\mu \sum_{m_S \in M} i(m_S|G)}{\sum_\theta \Pr(\theta) i(m_S|\theta)} \\
&= \sum_{m_S \in M} \left(\sum_{m_t \in \mathcal{M}_T} p_t^*(i, m_t) c^*(m_t|m_S) \right) \frac{\mu i(m_S|G) + (1-\mu) i(m_S|B)}{\sum_\theta \Pr(\theta) i(m_S|\theta)} \\
&\geq \sigma_t
\end{aligned}
$$

as required. $\square$

Sufficiency: Take any $Y \in \mathcal{Y}^*(t)$ such that, for all $m_S \in Y$ and m_T such that $c^*(m_t \mid m_S) > 0$, $p^*(i, m_t) \geq \sigma_t$; and for all $m_S \notin Y$ and m_t such that $c^*(m_t \mid m_S) > 0$, $p^*(i, m_t) < \sigma_t$.

Now consider the quadruple $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ defined as follows:

1. For receiver t, $i_t^* = i$, $c_t^* = c^*$, $p_t^* = p^*$ and $\boldsymbol{p}_{-t}^*$ is defined by (1), (2) and (3) using (p^*, c^*).

2. For receiver $r \neq t$, take any $Y_r \in \mathcal{Y}(r)$. Then, let $i_r^* = i$, p_r^* be defined such that if $m_S \in Y_r$ ($m_S \notin Y_r$), then $p_r(m_S|B)$ is the expected posterior of Sender conditional

on observing a message in Y_r (not in Y_r), c_r^* be defined such that for all $m_S \in Y$, $c_r^*(Y|m_S) = 1$ while for all $m_S \notin Y$, $c_r^*(\{m_S\}|m_S) = 1$, and $\boldsymbol{p}_{-r}^*$ are defined by (4), (1), (2) and (3) using (p_r^*, c_r^*).

3. t^* is defined by (8) using the previously defined quantities.

Notice here that for any non-targeted receiver $r \neq t$, Sender's reporting strategy and the target's belief are defined in an analogous way to that for receiver t. Hence, to verify that $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ constitutes an equilibrium, it suffices to show that for all $m_S \in M$ and each $m_t \in A(m_S)$, Sender's reporting strategy for receiver t, c_t^*, satisfies (7). To see this, take any such m_t in which $c^*(m_S|m_t) > 0$. If $m_S \in Y$, then $p^*(i, m_t) \geq \sigma_t$ so sending the message maximizes the probability the target buys. By a similar argument to Lemma 2, sending the message also maximizes the probability the target buys. We thus conclude that (7) holds. □

Proof of Lemma 9. Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be any equilibrium and $t \in \mathcal{R}$ any receiver. By (1) and (2), we see that for any non-target receiver $r \neq t$, $p_r^*(t \mid 1) = p_Y(1)$ and $p_r^*(t \mid 0) = p_Y(0)$. The claims in Lemma 9 are then easily verified by comparing $p_r^*(t \mid 1)$ and $p_r^*(t \mid 0)$ and $p_r^*(t \mid \emptyset)$ against a non-target r's level of skepticism σ_r. □

Proof of Lemma 10. Take any receiver $t \in \mathcal{R}$. By Lemma 8, every equilibrium in which Sender targets t can be associated to some $Y \in \mathcal{Y}(t)$ and vice versa, and under any equilibrium associated to Y, the target buys if and only if Sender observes $m_S \in Y$. This means that t buys the widget with probability $i(Y)$. Furthermore, by Lemma 9, the set of non-targets who buy upon observing the target take action $a_t \in \{0,1\}$ is $\{r \neq t : \sigma_t \leq p_Y(a_t)\}$, while the number of fans who buy upon not observing the target is $F - \mathbb{1}[t \leq F]$. Thus, the expected payoff from sales to non-targeted receivers is

$$\pi_t\bigg(i(Y)|\{r \neq t : \sigma_r \leq p_Y(1)\}| + (1 - i(Y))|\{r \neq t : \sigma_r \leq p_Y(1)\}|\bigg) + (1 - \pi_t)(F - \mathbb{1}[t \leq F]).$$

Adding these expressions together then yields the term inside the max operator of (19). Since Sender's payoff in the Sender-preferred equilibrium must be the maximum of these terms across all $Y \in \mathcal{Y}(t)$, this payoff is equal to (19). □

Proof of Lemma 11. Take any receiver $t \in \mathcal{R}$. First, suppose t is a skeptic. Then for all $Y \in \mathcal{Y}(t)$ in which $Y \neq \emptyset$, $p_Y(1) \geq \sigma_t$. Hence, the claim holds.

Next, suppose t is a fan. Take any $Y \in \mathcal{Y}(t)$ in which $Y \neq \{\emptyset, M\}$ and for all skeptics $r > F$, $p_Y(1) < \sigma_r$ holds. Then, Sender's equilibrium payoff is equal to

$$\begin{aligned}
& i(Y) + \pi_t\Big(i(Y)\{r \neq t : \sigma_r \leq p_Y(1)\} + (1 - i(Y))|\{r \neq t : \sigma_r \leq p_Y(0)\}\Big) \\
& \leq i(Y) + \pi_t\Big(i(Y)(F-1) + (1 - i(Y))(F-1)\Big) \\
& < 1 + \pi_t(F-1) \\
& \leq F.
\end{aligned}$$

The first inequality holds because, since no skeptic receiver buys upon observing that t buys, there is at most $F-1$ other receivers who buy upon observing t's action. The second (strict) inequality holds because, since $Y \neq \{\emptyset, M\}$, t buys with strictly interior probability. The final inequality holds as t's popularity is bounded above by 1. Hence, Sender is strictly better off in the equilibrium in which t always buys. I.e., when $Y = M$. $\square$

Proof of Proposition 4. By Proposition 1, Sender's (Sender-preferred) equilibrium payoff under information disclosure across all targets is the maximum of the payoff across all only fans equilibria and informative example equilibria. The payoff in any only-fans equilibrium is F. To compute the maximum payoff across all informative example equilibrium, we start by fixing a receiver $t \in \mathcal{R}$. Among all informative example equilibria in which t is targeted, Lemma 11 says that the sender-preferred one is the one with Y being maximum among all $Y \in \mathcal{Y}(t)$ in which the target's example is strong enough to convince some skeptic to buy, i.e., $Y \in \cup_{r>F}\mathcal{Y}(r)$. Thus, the payoff from (ii) is

$$\max_t \max_{Y : Y \in \mathcal{Y}(t) \cap \cup_{r>F}\mathcal{Y}(r)} \left[\begin{array}{c} i(Y) + \pi_t \left(\begin{array}{c} i(Y)\Big|\{r \neq t : \sigma_r \leq p_Y(1)\}\Big| + \\ (1 - i(Y))\Big|\{r \neq t : \sigma_r \leq p_Y(0)\}\Big| \end{array} \right) + \\ +(1-\pi_t)(F - \mathbb{1}[t \leq F]) \end{array} \right]. \tag{28}$$

Thus, if (20) holds, so that (28) is strictly less than F, the equilibrium is as described by Part 1 of Proposition 4. Analogously, if (21)) holds, so that (28) is strictly greater than F, then the equilibrium is as described by Part 2 of Proposition 4. $\square$

Proof of Corollary 2. Take any equilibrium $(t^*, \{i_t^*\}_{t \in \mathcal{R}}, \{c_t^*\}_{t \in \mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t \in \mathcal{R}})$ and let $t \in \mathcal{R}$

be any receiver. By Lemma 10, Sender's equilibrium expected payoff of targeting t is

$$\max_{Y\in\mathcal{Y}(t)}\left\{\begin{array}{c} i(Y)+\pi_t\Big(i(Y)\Big|\{r\neq t:\sigma_r\leq p_Y(1)\}\Big|+ \\ (1-i(Y))\Big|\{r\neq t:\sigma_r\leq p_Y(0)\}\Big|\Big)+(1-\pi_t)(F-\mathbb{1}[t\leq F]) \end{array}\right\}.$$

Hence a sufficient condition for Sender's payoff to be increasing in π_t is for

$$\pi_t\Big(i(Y)\Big|\{r\neq t:\sigma_r\leq p_Y(1)\}\Big|+(1-i(Y))\Big|\{r\neq t:\sigma_r\leq p_Y(0)\}\Big|\Big)-\pi_t(F-\mathbb{1}[t\leq F]) \tag{29}$$

to be increasing in π_t for all choices of Y. That is, (differentiating with respect to π_t):

$$\Big(i(Y)\Big|\{r\neq t:\sigma_r\leq p_Y(1)\}\Big|+(1-i(Y))\Big|\{r\neq t:\sigma_r\leq p_Y(0)\}\Big|\Big)-(F-\mathbb{1}[t\leq F])\geq 0.$$

Since this must hold for all Y, it suffices that it holds for the minimum, and rearranging gives the expression in Corollary 2. Similarly, if the maximum slope of (29) is ≤ 0, then the value of the target is decreasing with his popularity, as claimed. □

Proof of Proposition 5. First, notice that, by Lemma 7, Sender's value under Bayesian Persuasion of targeting t is $\mu/\sigma_t+\pi_t(t-1)\mu/\sigma_t+(1-\pi_t)F$.

Take any message $m_0\in M$. Consider the information structure i defined as follows: $i(m_0|G)=1, i(m_0|B)=(\mu(1-\sigma_t))/((1-\mu)\sigma_t), i(m_S|B)=(1-(\mu(1-\sigma_t))/((1-\mu)\sigma_t))/(|M|-1)$ for $m_S\neq m_0$. Since receiver r is less skeptical than the target t, $\{m_1\}\in\mathcal{Y}(r)$. Hence, by Lemma 10, under information disclosure, if Sender is endowed with information structure i, then Sender's preferred equilibrium payoff of targeting r is at least

$$\begin{aligned}
&i(\{m_0\})+\pi_r\Big(i(\{m_0\})|\{r'\neq r:\sigma_{r'}\leq p_{\{m_0\}}(1)\}|+(1-i(\{m_0\}))|\{r'\neq r:\sigma_{r'}\leq p_Y(1)\}|\Big)\\
&+(1-\pi_r)(F-\mathbb{1}[r\leq F])\\
&\quad=\mu/\sigma_r+\pi_r(t-1)\mu/\sigma_r+(1-\pi_r)F\\
&\quad\geq\mu/\sigma_t+\pi_r(t-1)\mu/\sigma_t+(1-\pi_r)F\\
&\quad>\mu/\sigma_t+\pi_t(t-1)\mu/\sigma_t+(1-\pi_t)F,
\end{aligned}$$

where the first inequality holds as r is less skeptical than t, and the second as r is strictly more popular than t and $\sigma_t<(t-1)\mu/F$. Thus, Sender is strictly better off under information disclosure than under Bayesian persuasion. □

Online Appendix
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B Nonlinear Sender's payoff and complementarities

Set-up. Suppose now that receiver i's payoff from buying now depends on other receivers' actions $a_{-i} \equiv (a_j)_{j \neq i}$, where $a_j = 1$ and $a_j = 0$ denote receiver j buying and not buying respectively, and is given by $u_i(a_{-i}, \theta)$. We assume that $u_i(a_{-i}, G) > u_i(a_{-i}, B)$, so a receiver has a stronger incentive to buy when the widget is good over bad, and that $u_i(a_{-i}, \theta)$ is non-decreasing in a_{-i}, so a receiver's incentive to buy is increasing in the number of other receivers who buy. We further assume that Sender's payoff given receivers' actions $a \equiv (a_i)_{i=1}^N$ is equal to $v(a)$, which is strictly increasing in a.

Equilibrium. With strategic complementarities, a receiver's purchase decision depends not only on her first-order beliefs about θ, but also on her higher-order beliefs about how others act. We extend the definition of an equilibrium to accommodate for the latter.

Formally, an equilibrium is a tuple $\{(t^*, i_t^*, c_t^*, p_t^*, p_{-t}^*, a_t^*, a_{-t}^*)\}_{t \in \mathcal{R}}$ under which

1. For each target $t \in \mathcal{R}$, the posteriors of non-targets $r \neq t$ about the state and the target's action $(p_r^*(t, o_t))_{r \neq t, o_t \in \{0,1,\emptyset\}}$, where $p_r^*(o_r, t) \in \Delta(\{G, B\} \times \{0, 1\})$ is the posterior given observation $o_r \in \{0, 1, \emptyset\}$, is derived using Bayes' rule, Sender's strategy, and the target's strategy. Formally, this requires that if $o_r \in \{0, 1\}$, so the non-target observes the target's action o_r, then the target's belief that the target's action is a_t and the quality is θ is

$$p_r^*(a_t, \theta | t, o_r) = \begin{cases} \dfrac{\sum_{\substack{m_t \in A(m_S): \\ a_t^*(m_t) = a_t}} \mu(\theta) \sum_{m_S \in M} i_t^*(m_S|\theta) c_t^*(m_t|m_S)}{\sum_{\substack{m_t \in A(m_S): \\ a_t^*(m_t) = a_t}} \sum_{\theta \in \{G,B\}} \mu(\theta) \sum_{m_S \in M} i_t^*(m_S|\theta) c_t^*(m_t|m_S)}, & a_t = o_r \\ 0, & a_t \neq o_r \end{cases} \tag{30}$$

Otherwise, i.e., $o_r = \emptyset$, so the receiver observes nothing, then

$$p_r^*(a_t, \theta | t, \emptyset) = \sum_{\substack{m_t \in A(m_S): \\ a_t^*(m_t) = a_t}} \mu(\theta) \sum_{m_S \in M} i_t^*(m_S|\theta) c_t^*(m_t \mid m_S), \qquad \forall (a_t, \theta). \tag{31}$$

2. For each target $t \in \mathcal{R}$, non-target receivers' play the largest continuation Bayes-Nash

equilibrium induced by the Sender targeting t. That is, let $A_t^*((p_r^*(t, o_t))_{r\neq t, o_t\in\{0,1,\emptyset\}})$ denote the set of Bayes-Nash equilibrium from targeting t. That is, the collection of strategies $(a_r(t))_{r\neq t}$, where $a_r(o_r, t)$ is the strategy of non-target r given observation o_r, that satisfy the following

$$a_r(o_r|t) \in \arg\max_{a_r\in\{0,1\}} a_r \sum_{(a_t,\theta)} \sum_{o_{-r,t}} \Big(u_r((a_t, a_{-r,t}(o_{-r,t})), \theta) \Big) Q_r(o_{-t,r}|a_t) p_r^*(a_t, \theta|t, o_t), \quad (32)$$

$$\forall o_{r,t} \in \{0, 1, \emptyset\}, t \neq r,$$

where $o_{-r,t} \equiv (o_{r'})_{r'\neq r,t} \in \{0, 1, \emptyset\}^{R-2}$ denote a vector of observations for non-r non-targets, and $Q_r(\cdot|a_t) \in \Delta\{a_t, \emptyset\}^{R-2}$ is the distribution over non-r non-targets' observations conditional on the target choosing action a_t.[18] Then, non-targets' strategies $(a_r^*(t))_{r\neq t}$ is the largest element of $A_t^*((p_r^*(t, o_t))_{r\neq t, o_t\in\{0,1,\emptyset\}})$. Because the game between non-targets is supermodular, the results of Milgrom & Roberts (1990) imply that such an element exists and is unique.

3. For each target $t \in \mathcal{R}$, his posterior p_t^* is derived using Bayes' rule and Sender's strategy. That is, it satisfies conditions (4) and (5) in the main text.

4. For each target $t \in \mathcal{R}$, his strategy a_t^* is the largest strategy which maximizes his payoff given the equilibrium played by non-targets $(a_r^*(t))_{r\neq t}$ and his own belief p_t^*. That is, for all m_t Sender communicates with positive probability, letting $Q_t(\cdot|a_t) \in \{a_t, \emptyset\}^{R-1}$ denote the distribution over non-targets' observations $o_{-t} \equiv (o_r)_{r\neq t}$ conditional on the target choosing action a_t,

$$a_t^*(m_t) = \max \arg\max_{a_t\in\{0,1\}} a_t \sum_{\theta} p_t^*(\theta|m_t) \sum_{o_t} u((a_t, a_{-t}^*(o_{-t}|t), \theta) Q_t(o_{-t}|a_t). \quad (33)$$

5. For each target $t \in \mathcal{R}$, Sender's choice of information structure i_t^* and communication strategy c_t^* are optimal given the posterior beliefs of the target p_t^*, and of non-targeted receivers $\boldsymbol{p}_{-t}^*$. That is,

$$i_t^* \in \arg\max_{i\in I} \sum_{\theta} \mu(\theta) \sum_{m_S} i(m_S|\theta) \sum_{m_t} c_t^*(m_t|m_S) \sum_{o_{-t}} v((a_t^*(m_t), a_{-t}^*(o_{-t}|t)) Q_t(o_{-t}|a_t^*(m_t)) \quad (34)$$

[18] This is the binominal distribution generated by a success probability, i.e., observing a_t, of π_t. We may also think of $Q_r(\cdot|a_t)$ as a distribution over $\{0, 1, \emptyset\}$ which assigns probability zero to the single observation $o_t \neq a_t, \emptyset$.

and, for each observed message $m_S \in \text{supp}(i_t^*(\cdot|\theta))$ and each allowable message $m_t \in A(m_S)$,

$$c_t^*(m_t|m_S) > 0 \Rightarrow m_t \in \underset{m_t \in A(m_S)}{\arg\max} \sum_{o_{-t}} v((a_t^*(m_t), a_{-t}^*(o_{-t}|t))Q_t(o_{-t}|a_t^*(m_t)). \tag{35}$$

Example Unravelling Theorem. We now work towards proving the Example Unravelling Theorem.

We begin by establishing an analogue to Lemma 1—The Power of Examples—in the main text. It says that for each non-target receiver, their belief that the quality is high is higher conditional on observing the target buy than not buy.

Lemma 1B (The power of examples)**.** *In any equilibrium $\{(t^*, i_t^*, c_t^*, p_t^*, \boldsymbol{p}_{-t}^*, a_t^*, a_{-t}^*)\}_{t\in\mathcal{R}}$ and for each receiver $t \in \mathcal{R}$, if Sender targets t, then receiver r's posterior belief is greater when he observes that t buys the widget than when he observes that t does not: for all $r \neq t$, $p_r^*(1, G|t, 1) \geq p_r^*(0, G|t, 1)$.*

Proof. To prove this claim, we will show that the target's strategy has a single-crossing property: there exists $\overline{p} \in [0, 1]$ such that $a_t^*(m_t) = 1$ if $p_t^*(m_t) \geq \overline{p}$, and $a_t^*(m_t) = 0$ otherwise. Equilibrium consistency, i.e., equation (30), then implies the claim.

Take any message m_t under which $a_t^*(m_t) = 1$, and any message m_t' under which $p_t^*(m_t') > p_t^*(m_t)$. Then,

$$\begin{aligned}
&\sum_\theta p_t^*(\theta|m_t') \sum_{o_{-t}} u((a_t, a_{-t}^*(o_{-t}|t), \theta)Q_t(o_{-t}|1) \\
&> \sum_\theta p_t^*(\theta|m_t) \sum_{o_{-t}} u((a_t, a_{-t}^*(o_{-t}|t), \theta)Q_t(o_{-t}|1) \geq 0,
\end{aligned}$$

where the first inequality holds as $u((a_t, a_{-t}, G) > u_t(a_{-t}, B)$ for all a_{-t}, and the second inequality holds as, by the definition of an equilibrium, the target's expected payoff from buying under message m_t must be non-negative. By (33), this means $a_t^*(m_t') = 1$. □

Next, given a set of strategies $(a_r(t))_{r\neq t}$ for non-targets and a targets' action a_t, let $F_t(\cdot|a_t, (a_r(t))_{r\neq t})$ denote the distribution over non-targets' actions $(a_r)_{r\neq t} \in A_{-t}$ conditional on the target taking action a_t. This is defined by

$$F_t((a_r)_{r\neq t}|a_t, (a_r(t))_{r\neq t})) \equiv Q_t(\{o_t : a_r(o_{r,t}|t) = a_r \forall r \neq t\}|a_t). \tag{36}$$

The next result – an analogue to Lemma 2 from the main text – says that the distribution over non-targets' actions $F_t(\cdot|a_t, (a_r(t))_{r\neq t})$ is stochastically higher conditional on the target buying over not buying.

Lemma 2B (Examples compel)**.** *In any equilibrium $\{(t^*, i_t^*, c_t^*, p_t^*, \boldsymbol{p}_{-t}^*, a_t^*, a_{-t}^*)\}_{t\in\mathcal{R}}$ and for each receiver $t \in \mathcal{R}$, if Sender targets t, then the distribution over the number of non-target receivers who buy the widget is stochastically higher if the target buys the widget: $F_t(\cdot|1, (a_r^*(t))_{r\neq t}))$ first-order stochastically dominates $F_t(\cdot|0, (a_r^*(t))_{r\neq t}))$.*

Proof. Let $\tau : \{0, \emptyset\}^{R-2} \to \{1, \emptyset\}^{R-2}$ denote the function defined such that for each $o_{-t} \in \{0, \emptyset\}^{R-2}$, $\tau(o_{-t})$ is obtained by replacing each observation $o_r = 0$ with $o_r = 1$, and keeping any $o_{r'} = \emptyset$ unchanged. Note that as the probability that each non-target observes the target's action is independent of the actual action taken by the target, $Q_r(\cdot|0) = Q_r(\tau(\cdot)|1)$. Hence, by (36),

$$F_t((a_r)_{r\neq t})|0, (a_r^*(t))_{r\neq t}))) = Q_t(\{o_t : a_r^*(o_{r,t}|t) = a_r \forall r \neq t\}|0) \tag{37}$$

$$F_t((a_r)_{r\neq t})|1, (a_r^*(t))_{r\neq t}))) = Q_t(\{o_t : a_r^*(\tau(o_{r,t})|t) = a_r \forall r \neq t\}|0). \tag{38}$$

By comparing (37) and (38), we see that if each non-target's strategy satisfies $a_r^*(1|t) \geq a_r^*(0|t)$, then $F_t(\cdot|1, (a_r^*(t))_{r\neq t})))$ would FOSD $F_t(\cdot|0, (a_r^*(t))_{r\neq t})))$, which yields Lemma 2B. We prove this next.

We begin by describing how each receivers' strategy is derived. Recall, by the definition of an equilibrium, that $(a_r^*(t))_{r\neq t}$ coincides with the largest equilibrium in the game between non-targets. By Milgrom and Roberts (1990), the largest equilibrium can be identified via an IESDS process. Formally, for each non-target receiver $r \neq t$, let O_r^1 denote the set of observations under which the non-target receiver r strictly prefers to not buy over buy assuming all other non-targets $r' \neq r, t$ buy regardless of their observation. Now, let O_r^2 denote the set of observations under which receiver r strictly prefers not to buy assuming all other non-targets r buy if and only if they observe $o_r \notin O_j^1$. Note that $O_r^2 \supseteq O_r^1$. Repeating this process thus yields a sequence of non-decreasing sets $O_r^1 \subseteq O_r^2 \subseteq O_r^3 \subseteq \cdots$ for each receiver. Letting $O_r \equiv \lim_{k\to\infty} O_r^k$, the argument of Milgrom and Roberts implies that in the largest equilibrium, a receiver buys if and only if they observe $o_j \notin O_j$.

We first show that for all non-targets $r \neq t$, $a_r^*(1|1) \geq a_r^*(0|t)$. By contradiction, suppose that for some $r \neq t$, $a_r^*(1|t) = 0$ but $a_r^*(0|t) = 1$. Let $\mathcal{R}'$ denote the set of such receivers, and k^* be the smallest round in which some receiver $r^* \in \mathcal{R}$ switches to not buy. The goal is to show at round k^*, receiver r^* strictly prefers to switch to not buying upon observing $o_r = 0$, which is a contradiction.

There are two possibilities to consider. First, suppose $k^* = 1$. As discussed above, receiver r^*'s round-1 strategy is defined under the belief that all other non-target buy regardless of their observation. Consider then receiver r^* payoff at round one from buying upon observing the target not buy. This is given by

$$\begin{pmatrix} u_{r^*}((0,\mathbf{1}),G)p^*_{r^*}(0,G|t,0) \\ +\sum_\theta u_{r^*}((0,\mathbf{1}),B)p^*_{r^*}(0,B|t,0) \end{pmatrix} \leq \begin{pmatrix} u_{r^*}((1,\mathbf{1}),G)p^*_{r^*}(0,G|t,0) \\ +\sum_\theta u_{r^*}((1,\mathbf{1}),B)p^*_{r^*}(0,B|t,0) \end{pmatrix}$$
$$\leq \begin{pmatrix} u_{r^*}((1,\mathbf{1}),G)p^*_{r^*}(1,G|t,1) \\ +\sum_\theta u_{r^*}((1,\mathbf{1}),B)p^*_{r^*}(1,B|t,1) \end{pmatrix} < 0$$

where the first inequality holds due to strategic complementarities, the second as, by Lemma 1B, $p^*_r(1,G|t,1) \geq p^*_r(0,G|t,0)$ and $u((a_t,a_{-t},G) > u_t(a_{-t},B)$ for all a_{-t}, and the last as, by definition, receiver r^* strictly prefers not to buy at round 1 conditional on observing the target buy. But this means that receiver r^*'s round 1 strategy conditional on observing the target not buy must be to not buy, which contradicts $a^*_r(0|t)=1$.

Now consider $k^*>1$. Let $a^{k^*-1}_{r'}(\cdot)$ denote the strategy for a non-r^* non-target receiver $r' \neq r,t$ at the k^*-1th round of the IESDS process. That is, $a^{k^*-1}_{r'}(o_{r'})=1$ if and only if $o_{r'} \in O^{k^*-1}_{r'}$. By the definition of k^*-1, all non-target receivers who observe the target buy will buy at round k^*-1: $a^{k^*-1}_{r'}(1)=1$. Therefore, letting $a^{k^*-1}_{-r^*,t}(o_{-r^*,t}) \equiv (a^{k^*-1}_{r'}(o_{r'}))_{r'\neq r,t}$, it follows that for any $o_{-r^*,t} \in \{0,\emptyset\}$, $a^{k^*-1}_{-r^*,t}(o_{-r^*,t}) \leq a^{k^*-1}_{-r^*,t}(\tau(o_{-r^*,t}))$. Therefore, for each state $\theta \in \Theta$, the expected payoff of receiver r^* at round k^*-1 from buying, conditional on observing the target not buy, satisfies

$$\begin{aligned}
&\sum_{o_{-t,r^*}} \left(u_{r^*}((0,a^{k^*-1}_{-r^*,t}(o_{-r^*,t})),\theta)\right)Q(o_{-r^*,t}|0) \\
&\leq \sum_{o_{-t,r^*}} \left(u_{r^*}((0,a^{k^*-1}_{-r^*,t}(\tau(o_{-r^*,t}))),\theta)\right)Q(\tau(o_{-r^*,t})|0) \\
&= \sum_{o_{-t,r^*}} \left(u_{r^*}((0,a^{k^*-1}_{-r^*,t}(o_{-r^*,t})),\theta)\right)Q(o_{-r^*,t}|1) \\
&\leq \sum_{o_{-t,r^*}} \left(u_{r^*}((1,a^{k^*-1}_{-r^*,t}(o_{-r^*,t})),\theta)\right)Q(o_{-r^*,t}|1),
\end{aligned}$$

where the last inequality holds due to strategic complementarities. It then follows that the expected payoff of receiver r^* at round k^*-1 from buying, conditional on observing the target not buy, satisfies

$$
\begin{aligned}
&\left(\begin{array}{c}
\sum_{o_{-r^*,t}}\left(u_{r^*}((0,a^{k^*-1}_{-r^*,t}(o_{-r^*,t})),G)\right)Q(o_{-r^*,t}|0)p^*_{r^*}(0,G|t,0)\\
+\sum_{o_{-r^*,t}}\left(u_{r^*}((0,a^{k^*-1}_{-r^*,t}(o_{-r^*,t})),B)\right)Q(o_{-r^*,t}|0)p^*_{r^*}(0,B|t,0)
\end{array}\right)\\
\leq &\left(\begin{array}{c}
\sum_{o_{-r^*,t}}\left(u_{r^*}((1,a^{k^*-1}_{-r^*,t}(o_{-r^*,t})),G)\right)Q(o_{-r^*,t}|1)p^*_{r^*}(0,G|t,0)\\
+\sum_{o_{-r^*,t}}\left(u_{r^*}((1,a^{k^*-1}_{-r^*,t}(o_{-r^*,t})),B)\right)Q(o_{-r^*,t}|1)p^*_{r^*}(0,B|t,0)
\end{array}\right)\\
\leq &\left(\begin{array}{c}
\sum_{o_{-r^*,t}}\left(u_{r^*}((1,a^{k^*-1}_{-r^*,t}(o_{-r^*,t})),G)\right)Q(o_{-r^*,t}|1)p^*_{r^*}(1,G|t,1)\\
+\sum_{o_{-r^*,t}}\left(u_{r^*}((1,a^{k^*-1}_{-r^*,t}(o_{-r^*,t})),B)\right)Q(o_{-r^*,t}|1)p^*_{r^*}(1,B|t,1)
\end{array}\right)<0,
\end{aligned}
$$

where the first two inequalities hold by a similar reasoning to the case where $k^* = 1$, and the third inequality as, by assumption, receiver r^* switches to not buy conditional on observing the target buy at round k^*. But this means that receiver r^*'s round k^* best-response conditional on observing the target not buy must be to not buy. This contradicts $a^*_r(0|t) = 1$. $\square$

We are now ready to prove the Example Unravelling Theorem.

Theorem 1B (Example-unravelling). *In any equilibrium $\{(t^*, i^*_t, c^*_t, p^*_t, p^*_{-t}, a^*_t, a^*_{-t})\}_{t\in\mathcal{R}}$, and for each receiver $t \in \mathcal{R}$,*

$$i^*_t \in \arg\max_{i\in I} \sum_{\theta} \Pr(\theta) \sum_{m_S} i(m_S \mid \theta) \sum_{m_t} c^*_t(m_t \mid m_S)\mathbb{1}[p^*_t(i, m_t) \geq \sigma_t] \tag{39}$$

and, for each observed message $m_S \in \operatorname{supp}(i^*_t(\cdot \mid \theta))$ and each allowable message $m_t \in A(m_S)$,

$$c^*_t(m_t \mid m_S) > 0 \Rightarrow m_t \in \arg\max_{m\in A(m_S)} \mathbb{1}[p^*_t(i^*_t, m) \geq \sigma_t]. \tag{40}$$

Proof. The logic is identical to the proof of the Example Unravelling Theorem in the main text. First, observe that in equilibrium, Sender's expected payoff from sending message m_t can be written as

$$\sum_{o_{-t}} v((a^*_t(m_t), a^*_{-t}(o_{-t}|t))Q_t(o_{-t}|a^*_t(m_t)) = V(a^*(m_t))$$

where

$$V(a_t) \equiv \sum_{a_{-t}} v((a_t, a_{-t}))F_t(a_{-t}|a_t, (a_r^*(t))_{r\neq t}))$$

Because $v(a)$ is strictly increasing in a, Lemma 1B implies that $V(a_t)$ is strictly increasing in the target's action a_t. Therefore, as in the main text, to establish that the Example Unravelling Theorem holds, it suffices to show that

$$\arg\max_{m_t \in A(m_S)} V(a_t^*(m_t)) \subseteq \arg\max_{m \in A(m_S)} \mathbb{1}[p_t^*(i_t^*, m) \geq \sigma_t]$$

To show this, take any message observed by Sender $m_S \in M$, and any message $m_t \in \arg\max_{m_t \in A(m_S)} V(a_t^*(m_t))$. If every allowable message $m_t \in A(m_S)$ sent by Sender induces the target not to buy, then $V(a_t^*(m_t)) = V(0)$ for all $m_t \in A(m_S)$. Hence, both sides of the above coincide.

Next, suppose there exists an allowable message $\hat{m}_t \in A(m_S)$ which induces the target to buy: $a^*(\hat{m}_t) = 1$. Take any $m_t \in \arg\max_{m_t \in A(m_S)} V(a_t^*(m_t))$. Then, $V(a_t^*(m_t)) \geq V(a_t^*(\hat{m}_t)) = V(1)$. Since $V(a_t)$ is strictly increasing in a_t, it follows that $a^*(m_t) = 1$, so $m_t \in \arg\max_{m \in A(m_S)} \mathbb{1}[p_t^*(i_t^*, m) \geq \sigma_t]$. □

Strategic complementarities and the double edged sword of popularity. We now discuss how strategic complementarities sharpen the double-edged sword of popularity. For simplify, we assume that a receiver's payoff depends only on (and is increasing in)the aggregate number of other receivers who buy the widget:

$$u_r(a_{-r}, \theta) = u_r\left(\sum_{r' \neq r} a_{r'}, \theta\right)$$

For each non-target $r \neq t$, note that we may define a CDF $F_r^*(\cdot \mid a_t)$ which captures the distribution over aggregate purchases by other non-targets, conditional on the target's action $a_t \in \{0, 1\}$:

$$F_r^*(n|a_t) \equiv Q_r\left(\left\{o_{-r,t} : \sum_{o_{-r,t}} a_{-r,t}^*(o_{-r,t}) \leq n\right\} \mid a_t\right), \qquad \forall n \in [0, R-2]$$

Then, the payoff to a non-target from buying, conditional on observing the target buy, is

$$\sum \big(u_r(1+n, G)p_r^*(1, G \mid t, 1) + u_r(1+n, B)p_r^*(1, B \mid t, 1)\big), dF_r^*(n \mid 1), \tag{41}$$

and, conditional on observing the target not buy, is

$$\sum \left(u_r(n,G)p_r^*(0,G \mid t,0) + u_r(n,B)p_r^*(0,B \mid t,0)\right), dF_r^*(n \mid 0). \tag{42}$$

By definition, a higher popularity for the target raises the probability that a non-target observes the target's action. Following the proof strategy of Lemma 2B, we may further show that a non-target's equilibrium action is higher conditional on observing the target buy than on observing nothing. Thus, a higher popularity increases $F_r^*(n \mid 1)$ in the FOSD order, i.e., other non-targets are more likely to buy, and so by (41) raises the incentive to buy when the target buys. By a similar logic, a higher popularity lowers the incentive to buy when the target does not buy. Hence, we obtain the following.

Remark 1. *In any equilibrium, a higher popularity for the target popularity increases the probability a non-target buys conditional on observing the target buy, and decreases the probability a non-target buys conditional on observing the target not buy*

Remark 1 is the sense in which strategic complementarities sharpen the double-edged sword of popularity. When the target buys (does not buy), a higher popularity raises (lowers) the probability that a non-target buys by simultaneously increasing the likelihood the non-target observes the target's action and, conditional on doing so, raises (lowers) each non-target's belief that others also buy. The latter effect is present only in a setting with strategic complementarities.

C Multiple Targets

We now consider the case where Sender can chooses a subset of receivers to target $\mathcal{T} \subseteq \mathcal{R}$ subject to $1 \leq |\mathcal{T}| \leq T$ for some $1 \leq T < R$.

Communication Protocols. We first extend the definition of a communication protocol to allow for multiple targets. Formally, a *communication protocol* is tuple $((M^r)_{r\in\mathcal{R}}, (A^r)_{r\in\mathcal{R}}, (I_\mathcal{T})_\mathcal{T})$. $M \equiv (M^r)_{r\in\mathcal{R}}$ comprises of a collection messages. $I_\mathcal{T} \subseteq \{i_\mathcal{T} : \Theta \to \Delta(M^\mathcal{T})\}$ is a collection of information structures available to Sender upon targeting $\mathcal{T}$, where $M^\mathcal{T} \equiv \times_{t\in\mathcal{T}} M^t$. Finally, for each $\boldsymbol{m}_S^\mathcal{T} \equiv (m_S^t)_{t\in\mathcal{T}} \in M^\mathcal{T}$, $\times_{t\in\mathcal{T}} A^t(m_S^t)$ is a set of allowable messages.

The communication protocol induces a communication game between Sender and the target $\mathcal{T}$ as follows. First, Sender chooses an information structure $i_\mathcal{T} \in I_\mathcal{T}$. Conditional on θ, a vector of messages $\boldsymbol{m}_S^\mathcal{T} \in M^\mathcal{T}$ is drawn with probability $i_\mathcal{T}(\boldsymbol{m}_S^\mathcal{T} \mid \theta)$, where the message m_S^t can be understood as the information that Sender acquires which can be communicated to the target $t \in \mathcal{T}$. Sender then chooses what acquired information to communicate to each targeted receiver, i.e., by choosing a $m^t \in A^t(m^t_S)$ for each $t \in \mathcal{T}$. Each targeted receiver $t \in \mathcal{T}$ privately observes Sender's communication to them, m^t, and then simultaneously decides whether to act. Finally, each non-targeted receiver independently observes at one most targeted receiver, where targeted receiver $t \in \mathcal{T}$ is observed with probability $\pi_{t|\mathcal{T}} > 0$ and $\sum_{t \in \mathcal{T}} \pi_{t|\mathcal{T}} \in (0, 1)$.

Observe that what Sender can communicate to target t, $A^t(m^t_S)$, depends only on the information Sender acquires for target t, m^t_S, and not on what information Sender acquires for other targets, $(m^{t'}_S)_{t' \in \mathcal{T}-t}$. However, the information that Sender can acquire for target t can depend on the information Sender acquires for other targets. It is through this channel that the information Sender communicates to each target ex-post can be correlated.

To make this point precise, let $I_{\mathcal{T}-t} \subseteq \{i_{\mathcal{T}-t} : \Theta \to \Delta(M^{\mathcal{T}-t})\}$ denote the set of all available marginal distribution over messages for non-t targeted receivers under some information structure in $I_{\mathcal{T}}$. This set captures the information Sender can acquire for non-t targets, and is defined by

$$I_{\mathcal{T}-t} \equiv \left\{ i_{\mathcal{T}-t} : \begin{array}{c} \exists i_{\mathcal{T}} \in I_{\mathcal{T}} \text{ s.t. } \forall \theta \in \Theta,\ \boldsymbol{m}^{\mathcal{T}-t}_S \in M^{\mathcal{T}-t}, \\ \sum_{m^t_S \in M^t} i_{\mathcal{T}}(m^t_S, \boldsymbol{m}^{\mathcal{T}-t}_S|\theta) = i_{\mathcal{T}-t}(\boldsymbol{m}^{\mathcal{T}-t}_S|\theta) \end{array} \right\}.$$

Furthermore, for each $i_{\mathcal{T}-t} \in I_{\mathcal{T}-t}$ and messages for non-t targets $\boldsymbol{m}^{\mathcal{T}-t}_S \in M^{\mathcal{T}-t}$, let $I_t(\boldsymbol{m}^{\mathcal{T}-t}_S \mid i_{\mathcal{T}-t}) \subseteq \{i_t(\cdot|\boldsymbol{m}^{\mathcal{T}-t}_S, \cdot) : \Theta \to \Delta(M^t)\}$ denote the subset of marginal distributions over receiver t's messages received by Sender on each state, conditional on $\boldsymbol{m}^{\mathcal{T}-t}$ being drawn, inducible by some information structure available to Sender. This captures the information Sender can acquire for target t conditional on the information already acquired for non-t targets, and is given by the set

$$\begin{aligned} I_t(\boldsymbol{m}^{\mathcal{T}-t}_S \mid i_{\mathcal{T}-t}) &\equiv \left\{ i_t(\cdot|\boldsymbol{m}^{\mathcal{T}-t}_S, \cdot) : \begin{array}{c} \exists i_{\mathcal{T}} \in I_{\mathcal{T}} \text{ s.t. } \forall \theta \in \Theta, \\ i_{\mathcal{T}}(m^t_S, \boldsymbol{m}^{\mathcal{T}-t}_S|\theta) = i_t(m^t_S|\boldsymbol{m}^{\mathcal{T}-t}_S, \theta) i_{\mathcal{T}-t}(\boldsymbol{m}^{\mathcal{T}-t}_S|\theta) \end{array} \right\} \\ &= \times_{\theta \in |\Theta} \underbrace{\left\{ i_t(\cdot|\boldsymbol{m}^{\mathcal{T}-t}_S, \theta) : \begin{array}{c} \exists i_{\mathcal{T}} \in I_{\mathcal{T}} \text{ s.t.} \\ i_{\mathcal{T}}(m^t_S, \boldsymbol{m}^{\mathcal{T}-t}_S|\theta) = i_t(m^t_S|\boldsymbol{m}^{\mathcal{T}-t}_S, \theta) i_{\mathcal{T}-t}(\boldsymbol{m}^{\mathcal{T}-t}_S|\theta) \end{array} \right\}}_{\equiv I_t(\boldsymbol{m}^{\mathcal{T}-t}_S | i_{\mathcal{T}-t}, \theta)} \end{aligned}$$

Notice then that fixing *how* Sender acquires information for non-t targets, $i_{\mathcal{T}-t}$, acquiring different information for non-t targets' may lead to different possibilities for acquiring information for target t: $\boldsymbol{m}^{\mathcal{T}-t}_S \neq \tilde{\boldsymbol{m}}^{\mathcal{T}-t}_S$ may imply $I_t(\boldsymbol{m}^{\mathcal{T}-t}_S \mid i_{\mathcal{T}-t}) \neq I_t(\tilde{\boldsymbol{m}}^{\mathcal{T}-t}_S \mid i_{\mathcal{T}-t})$. Likewise, fixing the information acquired for non-t targets, $\boldsymbol{m}^{\mathcal{T}-t}_S$, changing how Sender acquired information for other targets may lead to different possibilities for acquiring information for target t: $i_{\mathcal{T}-t} \neq i'_{\mathcal{T}-t}$ may imply $I_t(\boldsymbol{m}_S^{\mathcal{T}-t} \mid i_{\mathcal{T}-t}) \neq I_t(\boldsymbol{m}_S^{\mathcal{T}-t} \mid i'_{\mathcal{T}-t})$.

As it turns out, the multi-target Example Unravelling Theorem (to be stated) can be substantially strengthened when the information Sender can acquire for target t is independent of how and what information is acquired for non-t targets. We call such communication protocols **private**. Formally, let $I_{\mathcal{T}}(t) \subseteq \{i_t : \Theta \to \Delta M^t\}$ denote the subset of marginal distributions over messages Sender can acquire for Receiver t, i.e.,

$$I_{\mathcal{T}}(t) \equiv \left\{ i_t : \exists i_{\mathcal{T}} \in I_{\mathcal{T}} \text{ s.t.} \sum_{\boldsymbol{m}_S^{\mathcal{T}-t}} i_{\mathcal{T}}(m_S^t, \boldsymbol{m}_S^{\mathcal{T}-t}|\theta) = i_t(m_S^t|\theta), \quad \forall \theta, m_S^t \right\}$$

$$= \times_{\theta\in\Theta} \underbrace{\left\{ i_t(\cdot|\theta) : \exists i_{\mathcal{T}} \in I_{\mathcal{T}} \text{ s.t.} \sum_{\boldsymbol{m}_S^{\mathcal{T}-t}} i_{\mathcal{T}}(m_S^t, \boldsymbol{m}_S^{\mathcal{T}-t}|\theta) = i_t(m_S^t|\theta), \quad \forall m_S^t \right\}}_{\equiv I_{\mathcal{T}}(t|\theta)}$$

Then, a communication protocol is private if for all $\mathcal{T}$ and $t \in \mathcal{T}$, for all $i_{\mathcal{T}-t} \in I_{\mathcal{T}-t}$ and messages $\boldsymbol{m}_S^{\mathcal{T}} \in M^{\mathcal{T}}$, $I_t(\boldsymbol{m}_S^{\mathcal{T}-t} \mid i_{\mathcal{T}-t}) = I_{\mathcal{T}}(t)$ holds (equivalently, for all $\theta \in \Theta$, $I_t(\boldsymbol{m}_S^{\mathcal{T}-t} \mid i_{\mathcal{T}-t}, \theta) = I_{\mathcal{T}}(t|\theta)$ holds). This class includes, for example, the standard unconstrained information design problem introduced in (Bergemann and Morris, 2019).

Strategies and payoffs. A strategy for Sender is a triple $(\mathcal{T}, \{i_{\mathcal{T}}\}_{\mathcal{T}}, \{c_{\mathcal{T}}\}_{\mathcal{T}})$, where $\mathcal{T}$ is the Sender's choice of target, and, for each possible $\mathcal{T}$, $i_{\mathcal{T}} \in I^{\mathcal{T}}$ is Sender's choice of information structure, and $c_{\mathcal{T}} \equiv (c_{t|\mathcal{T}})$ is a collection of communication strategies for Sender, where

$$c_{t|\mathcal{T}} \in C^t \equiv \{c_t : M^t \to \Delta(2^{M^t}) \mid \forall m_S^t \in M^t, \ \text{supp}(\tilde{c}^t(\cdot|m_S^t)) \subseteq A^t(m_S^t)\}$$

is Sender's communication strategy for each possible message acquired for receiver t. Notice here that we focus on communication strategies for Sender in which the messages she sends to a target t depends only on the information she acquires for target t, m^t, and not of the information she acquires that can be communicated to other non-t targets.[19]

If a receiver $r \in \mathcal{R}$ is targeted, so $r = t \in \mathcal{T}$, then his posterior belief $p_t(i_{\mathcal{T}}, m_t)$ depends on Sender's information structure $i_{\mathcal{T}}$, and observed message m^t. Meanwhile, the posterior of a receiver who is not a target $r \notin \mathcal{T}$, $p_r(\mathcal{T} \mid t, o_t)$, depends on the target $\mathcal{T}$, the receiver she observes $t \in \mathcal{T} \cup \{\emptyset\}$ (where $t = \emptyset$ means she observes no target), and the targeted receiver's action if observed, $o_t \in \{0, 1\}$.

[19]This rules out signalling equilibria in which Sender changes how she reports her acquired information to a target purely to signal that she has acquired different information for other targets.

Given the above, Sender's payoff from targeting $\mathcal{T}$ and sending message $\boldsymbol{m}^{\mathcal{T}}$ given beliefs $(\boldsymbol{p}_{\mathcal{T}}, \boldsymbol{p}_{-\mathcal{T}})$ is

$$
\begin{aligned}
V_{\mathcal{T},i_{\mathcal{T}}}(\boldsymbol{m}^{\mathcal{T}}, \boldsymbol{p}_{\mathcal{T}}, \boldsymbol{p}_{-\mathcal{T}}) \equiv & \sum_{t \in \mathcal{T}} \left(\mathbb{1}[p_t(i_{\mathcal{T}}, m^t) \geq \sigma_t] \left(1 + \pi_{t|\mathcal{T}} \sum_{r \notin \mathcal{T}} \mathbb{1}[p_r(\mathcal{T} \mid t, 1) \geq \sigma_r] \right) \right) \\
& + \sum_{t \in \mathcal{T}} \left(\mathbb{1}[p_t(i_{\mathcal{T}}, m^t) < \sigma_t] \left(\pi_{t|\mathcal{T}} \sum_{r \notin \mathcal{T}} \mathbb{1}[p_r(\mathcal{T} \mid t, 0) \geq \sigma_r] \right) \right) \\
& + \left(1 - \sum_{t \in \mathcal{T}} \pi_{t|\mathcal{T}} \right) \sum_{r \notin \mathcal{T}} \mathbb{I}[p_r(\mathcal{T}|\emptyset) \geq \sigma_r]
\end{aligned}
$$

Because Sender's communication (strategy) to a target depends only on the information she acquires for that target, the target's equilibrium belief varies only with Sender's information acquisition for the target, and not with her information acquisition for other targets. Thus, fixing a target t, the belief of any other target $t' \in \mathcal{T} - t$ depends solely on the Sender's information acquisition for non-t targets, $i_{\mathcal{T}-t}$, and can thus be written as $p_{t'}(i_{\mathcal{T}-t}, m^{t'})$. By contrast, t's posterior depends both on $i_{\mathcal{T}-t}$ and on the Sender's information acquisition for t conditional on the information received for non-t targets,

$$
i_{t|\mathcal{T}} \equiv \{ i_t(\cdot \mid \boldsymbol{m}_S^{\mathcal{T}-t}, \cdot) \}_{\boldsymbol{m}_S^{\mathcal{T}-t} \in M^{\mathcal{T}-t}},
$$

so it can be written as $p_t((i_{\mathcal{T}-t}, i_{t|\mathcal{T}}), m^t)$. This allows us to express Sender's payoff in a form that isolates the effect of changes in how Sender acquires information for target t as follows:

$$
\begin{aligned}
V_{\mathcal{T},i_{\mathcal{T}}}(\boldsymbol{m}^{\mathcal{T}}, \boldsymbol{p}_{\mathcal{T}}, \boldsymbol{p}_{-\mathcal{T}}) \equiv & \sum_{t' \in \mathcal{T}-t} \left(\mathbb{1}[p_{t'}(i_{\mathcal{T}-t}, m^{t'}) \geq \sigma_{t'}] \left(1 + \pi_{t'|\mathcal{T}} \sum_{r \notin \mathcal{T}} \mathbb{1}[p_r(\mathcal{T} \mid t', 1) \geq \sigma_r] \right) \right) \\
& + \sum_{t' \in \mathcal{T}-t} \left(\mathbb{1}[p_{t'}(i_{\mathcal{T}-t}, m^{t'}) < \sigma'_t] \left(\pi_{t'|\mathcal{T}} \sum_{r \notin \mathcal{T}} \mathbb{1}[p_r(\mathcal{T} \mid t', 0) \geq \sigma_r] \right) \right) \\
& + \left(1 - \sum_{t \in \mathcal{T}} \pi_{t|\mathcal{T}} \right) \sum_{r \notin \mathcal{T}} \mathbb{I}[p_r(\mathcal{T}|\emptyset) \geq \sigma_r] \\
& + \left(\mathbb{1}[p_t((i_{\mathcal{T}-t}, i_{t|\mathcal{T}}), m^t) \geq \sigma_t] \left(1 + \pi_{t|\mathcal{T}} \sum_{r \notin \mathcal{T}} \mathbb{1}[p_r(\mathcal{T} \mid t, 1) \geq \sigma_r] \right) \right) \\
& + \left(\mathbb{1}[p_t p_t((i_{\mathcal{T}-t}, i_{t|\mathcal{T}}), m^t) < \sigma_t] \left(\pi_{t|\mathcal{T}} \sum_{r \notin \mathcal{T}} \mathbb{1}[p_r(\mathcal{T} \mid t, 0) \geq \sigma_r] \right) \right)
\end{aligned}
$$

The first three terms, which is the payoff Sender obtains from non-t receivers assuming non-targets only observe the actions from targets in $\mathcal{T} - t$, does not depend on how

Sender acquires information for target t. Meanwhile, the last two terms, comprising of Sender's marginal gain from targeting receiver t, which includes both the direct benefit from sometimes persuading t to buy, and the indirect effect that observing t's actions has on non-targets' actions, do.

Equilibrium. An assessment $(\mathcal{T}, \{i_{\mathcal{T}}\}_{\mathcal{T}}, \{c_{\mathcal{T}}\}_{\mathcal{T}}, \{\boldsymbol{p}_{\mathcal{T}}, \boldsymbol{p}_{-\mathcal{T}}\}_{\mathcal{T}})$ is an equilibrium if:

1. For each target $\mathcal{T}$, the posterior $p_r(\mathcal{T})$ of each non-targeted receiver $r \notin \mathcal{T}$ is derived using Bayes' rule, Sender's strategy, and the targets' posteriors, $\boldsymbol{p}_{\mathcal{T}}$:

$$p_r(\mathcal{T} \mid t, 1) = \frac{Pr(\theta = G) \sum_{\boldsymbol{m}_S^{\mathcal{T}} \in M^{\mathcal{T}}} \sum_{\substack{\boldsymbol{m}^{\mathcal{T}} \in \times_{t \in \mathcal{T}} A_t(m_S^t) \\ p_t(i_{\mathcal{T}}, m^t) \geq \sigma_t}} i_{\mathcal{T}}(\boldsymbol{m}_S^{\mathcal{T}} | \theta = G) c_t(m^t | m_S^t)}{\sum_{\theta \in \{G,B\}} \Pr(\theta) \sum_{\boldsymbol{m}_S^{\mathcal{T}} \in M^{\mathcal{T}}} \sum_{\substack{\boldsymbol{m}^{\mathcal{T}} \in \times_{t \in \mathcal{T}} A_t(m_S^t) \\ p_t(i_{\mathcal{T}}, m^t) \geq \sigma_t}} i_{\mathcal{T}}(\boldsymbol{m}_S^{\mathcal{T}} | \theta) c_t(m^t | m_S^t)},$$

$$p_r(\mathcal{T} \mid t, 0) = \frac{Pr(\theta = G) \sum_{\boldsymbol{m}_S^{\mathcal{T}} \in M^{\mathcal{T}}} \sum_{\substack{\boldsymbol{m}^{\mathcal{T}} \in \times_{t \in \mathcal{T}} A_t(m_S^t) \\ p_t(i_{\mathcal{T}}, m^t) < \sigma_t}} i_{\mathcal{T}}(\boldsymbol{m}_S^{\mathcal{T}} | \theta = G) c_t(m^t | m_S^t)}{\sum_{\theta \in \{G,B\}} \Pr(\theta) \sum_{\boldsymbol{m}_S^{\mathcal{T}} \in M^{\mathcal{T}}} \sum_{\substack{\boldsymbol{m}^{\mathcal{T}} \in \times_{t \in \mathcal{T}} A_t(m_S^t) \\ p_t(i_{\mathcal{T}}, m^t) < \sigma_t}} i_{\mathcal{T}}(\boldsymbol{m}_S^{\mathcal{T}} | \theta) c_t(m^t | m_S^t)}. \tag{43}$$

if $t \neq \emptyset$, and

$$p_r(\mathcal{T} \mid \emptyset) = \mu, \tag{44}$$

otherwise.

2. For each target $\mathcal{T}$ and targeted receiver $t \in \mathcal{T}$, his posterior p_t is derived using Bayes' rule and Sender's strategy: for all m_t Sender communicates with positive probability:[20]

$$p_t(i_{\mathcal{T}}, m_t) = \frac{\mu \sum_{\boldsymbol{m}_S^{\mathcal{T}-t} \in M^{\mathcal{T}-t}} \sum_{m_S^t \in M^t} i_{t|\mathcal{T}}(m_S^t | \boldsymbol{m}_S^{\mathcal{T}-t}, \theta = 1) i_{\mathcal{T}-t}(\boldsymbol{m}_S^{\mathcal{T}-t} | \theta = 1) c_t(m^t \mid m_S^t)}{\sum_{\theta \in \{G,B\}} \Pr(\theta) \sum_{\boldsymbol{m}_S^{\mathcal{T}-t} \in M^{\mathcal{T}-t}} \sum_{m_S^t \in M^t} \sum_{m_S \in M} i_{t|\mathcal{T}}(m_S^t | \boldsymbol{m}_S^{\mathcal{T}-t}, \theta) i_{\mathcal{T}-t}(\boldsymbol{m}_S^{\mathcal{T}-t} | \theta) c_t(m^t \mid m_S^t)}, \tag{45}$$

$$= \frac{\mu \sum_{\boldsymbol{m}_S^{\mathcal{T}-t'} \in M^{\mathcal{T}-t'}} i_{\mathcal{T}-t'}(\boldsymbol{m}_S^{\mathcal{T}-t'} | \theta = G) c_t(m^t \mid m_S^t)}{\sum_{\theta \in \{G,B\}} \Pr(\theta) \sum_{\boldsymbol{m}_S^{\mathcal{T}-t'} \in M^{\mathcal{T}-t'}} i_{\mathcal{T}-t'}(\boldsymbol{m}_S^{\mathcal{T}-t'} | \theta) c_t(m^t \mid m_S^t)}, \forall t' \in \mathcal{T} - t. \tag{46}$$

[20] The first equivalence emphasizes that a target t's belief depends jointly on how Sender acquires information for non-t targets, $i_{\mathcal{T}-t}$, and how Sender acquires information for target t given the information acquired for non-t targets, $i_{t|\mathcal{T}}$. The second equivalence emphasizes that holding fixed how Sender acquires information for target t, changing how Sender acquires information for other non-t targets does not affect target t's beliefs. Put differently, for any other target $t' \neq t$, a target t's belief depends only on the distribution over non-t' targets' messages, $i_{\mathcal{T}-t'}$.

Meanwhile, if $m^t = \{\hat{m}^t_S\} \notin A_t(m^t_S)$ for all $m^t_S \neq \hat{m}^t_S$, then[21]

$$p_t(i_{\mathcal{T}}, \{m_S\}) = \frac{\mu\, i_t(m^t_S|\theta = G)}{\sum_{\theta\in\{G,B\}} \Pr(\theta) i_t(m^t_S|\theta)}. \tag{47}$$

where i_t is the marginal distribution over target t's messages observed by Sender.

3. For each target $\mathcal{T}$, Sender's choice of information structure $i_{\mathcal{T}}$ and communication strategies $c_{\mathcal{T}}$ are optimal given the posterior beliefs of the targets, $\boldsymbol{p}_{\mathcal{T}}$, and of non-targeted receivers, $\boldsymbol{p}_{-\mathcal{T}}$:

$$i_{\mathcal{T}} \in \arg\max_{i\in I_{\mathcal{T}}} \sum_{\theta} \Pr(\theta) \sum_{\boldsymbol{m}^{\mathcal{T}}_S} i(\boldsymbol{m}^{\mathcal{T}}_S \mid \theta) \sum_{\boldsymbol{m}^{\mathcal{T}}} \left(\prod_{t\in\mathcal{T}} c_{t|\mathcal{T}}(m^t \mid m^t_S)\right) V_{\mathcal{T},i}(\boldsymbol{m}^{\mathcal{T}}, \boldsymbol{p}_{\mathcal{T}}, \boldsymbol{p}_{-\mathcal{T}}) \tag{48}$$

and, for each observed message $\boldsymbol{m}^{\mathcal{T}}_S \in \mathrm{supp}(i_{\mathcal{T}}(\cdot \mid \theta))$, targeted receiver $t \in \mathcal{T}$ and each allowable message $m^t \in A(m^t_S)$,

$$c_{t|\mathcal{T}}(m^t \mid m^t_S) > 0 \Rightarrow m^t \in \arg\max_{\tilde{m}^t\in A_t(m^t_S)} V_{\mathcal{T},i_{\mathcal{T}}}((\tilde{m}^t, \boldsymbol{m}^{\mathcal{T}-t}), \boldsymbol{p}_{\mathcal{T}}, \boldsymbol{p}_{-\mathcal{T}}). \tag{49}$$

4. Sender's choice of target is sequentially optimal:

$$\mathcal{T} \in \arg\max_{\mathcal{T}'\subseteq\mathcal{R}:1\leq|\mathcal{T}|\leq T} \sum_{\theta} \Pr(\theta) \sum_{\boldsymbol{m}^{\mathcal{T}}_S} i_{\mathcal{T}}(\boldsymbol{m}^{\mathcal{T}}_S \mid \theta) \sum_{\boldsymbol{m}^{\mathcal{T}}} \left(\prod_{t\in\mathcal{T}} c_{t|\mathcal{T}}(m^t \mid m^t_S)\right) V_{\mathcal{T},i_{\mathcal{T}}}(\boldsymbol{m}^{\mathcal{T}}, \boldsymbol{p}_{\mathcal{T}}, \boldsymbol{p}_{-\mathcal{T}}). \tag{50}$$

Example Unravelling Theorem. We now set out to prove the multi-target version of the Example Unravelling Theorem. We begin with two analogues to our main-text observations. First, Lemma 1C says that given any targeted receiver $t \in \mathcal{T}$, holding fixed non-targets' observations / non-observations of non-t targets actions, a positive example by t – i.e., t buys the good – induces the non-target to hold more positive beliefs about the quality of the widget.

Lemma 1C (The power of examples)**.** *In any equilibrium $(\mathcal{T}^*, \{i^*_{\mathcal{T}}\}_{\mathcal{T}}, \{c^*_{\mathcal{T}}\}_{\mathcal{T}}, \{\boldsymbol{p}^*_{\mathcal{T}}, \boldsymbol{p}^*_{-\mathcal{T}}\}_{\mathcal{T}})$, and for each set of targets $\mathcal{T}$, if Sender targets $\mathcal{T}$, then for all targeted receivers $t \in \mathcal{T}$, receiver*

[21]This last requirement says that, if the communication protocol allows Sender to credibly communicate the message he observes, then the target's belief upon observing such credible communication must coincide with Sender's belief. This is also a version of the "no signaling what you don't know condition".

r's posterior belief is greater when he observes that t buys the widget than when he observes that t does not: for all $r \notin \mathcal{T}$, $p_r^*(\mathcal{T} \mid t, 1) \geq p_r^*(\mathcal{T} \mid t, 0)$.

Proof. The proof is similar to that for Lemma 1 in the main text. Let $Q(m_t|1)$ denote the distribution over messages for target t that induce t to buy and not buy respectively. Then, by (43) and (45),

$$p_r^*(\mathcal{T}|t,1) = \sum_{\substack{m_t \in M^t \\ p_t^*(i_{\mathcal{T}}, m_t) \geq \sigma_t}} p_t^*(i_{\mathcal{T}}, m_t) Q(m_t|1) \geq \sum_{\substack{m_t \in M^t \\ p_t^*(i_{\mathcal{T}}, m_t) \geq \sigma_t}} \sigma_t Q(m_t|1) = \sigma_t$$

while

$$p_r^*(\mathcal{T}|t,0) = \sum_{\substack{m_t \in M^t \\ p_t^*(i_{\mathcal{T}}, m_t) < \sigma_t}} p_t^*(i_{\mathcal{T}}, m_t) Q(m_t|1) \leq \sum_{\substack{m_t \in M^t \\ p_t^*(i_{\mathcal{T}}, m_t) < \sigma_t}} \sigma_t Q(m_t|1) = \sigma_t$$

so $p_r^*(\mathcal{T} \mid t, 1) \geq p_r^*(\mathcal{T} \mid t, 0)$ □

Next, Lemma 2C says that given any targeted receiver $t \in \mathcal{T}$, holding fixed non-targets' observations / non-observations of non-t targets actions, on average, a positive example from t raises the probability the non-target buys.

Lemma 2C (Examples compel). *In any equilibrium $(\mathcal{T}^*, \{i_{\mathcal{T}}^*\}_{\mathcal{T}}, \{c_{\mathcal{T}}^*\}_{\mathcal{T}}, \{\boldsymbol{p}_{\mathcal{T}}^*, \boldsymbol{p}_{-\mathcal{T}}^*\}_{\mathcal{T}})$, and for each set of targets $\mathcal{T}$, if Sender targets $\mathcal{T}$, then for all targeted receivers $t \in \mathcal{T}$, receiver r's probability of buying the widget is greater if the target buys the widget: for all $r \neq t$,*

$$\pi_{t|\mathcal{T}} \mathbb{1}[p_r^*(\mathcal{T} \mid t, 1) \geq \sigma_r] \geq \pi_{t|\mathcal{T}} \mathbb{1}[p_r^*(\mathcal{T} \mid t, 0) \geq \sigma_r]. \tag{51}$$

Proof. Follows immediately from Lemma 1C. □

We are now ready to state the weak version of the multi-target Example Unravelling Theorem. It says that holding fixed a targeted receiver t and any information received which is relevant for non-t targets, Sender's equilibrium communication must maximize the probability that t buys the widget. That is, after fixing her communication to non-t targets, Sender optimally communicates to target t as if no other receiver exists.

Theorem 1C (Weak Example-unravelling). *In any equilibrium $(\mathcal{T}^*, \{i_{\mathcal{T}}^*\}_{\mathcal{T}}, \{c_{\mathcal{T}}^*\}_{\mathcal{T}}, \{\boldsymbol{p}_{\mathcal{T}}^*, \boldsymbol{p}_{-\mathcal{T}}^*\}_{\mathcal{T}})$, for each set of targets $\mathcal{T}$, targeted receiver $t \in \mathcal{T}$, and all messages for non-t targets observed by*

Sender $\boldsymbol{m}_S^{\mathcal{T}-t} \in M^{\mathcal{T}-t}$ with strictly positive probability,

$$i^*_{t|\mathcal{T}}(\cdot|\boldsymbol{m}_S^{\mathcal{T}-t},\cdot) \in \underset{i_{t|\mathcal{T}}(\cdot|\boldsymbol{m}_S^{\mathcal{T}-t},\cdot)\in I_t(\boldsymbol{m}_S^{\mathcal{T}-t}|i^*_{\mathcal{T}-t})}{\arg\max} \sum_\theta \Pr(\theta) \sum_{m_S^t} i_{t|\mathcal{T}}(m_S^t|\boldsymbol{m}_S^{\mathcal{T}-t},\theta) \sum_{m^t} c^*_{t|\mathcal{T}}(m^t \mid m_S^t) \mathbb{1}[p_t^*(i^*_{\mathcal{T}}, m^t) \geq \sigma_t]. \tag{52}$$

Additionally, for each observed message $m_S^t \in$ supp$(i_t(\cdot \mid \theta))$ and each allowable message $m^t \in A_t(m_S^t)$,

$$c^*_{t|\mathcal{T}}(m^t \mid m_S^t) > 0 \Rightarrow m^t \in \underset{m\in A(m_S)}{\arg\max}\, \mathbb{1}[p_t^*(i^*_{\mathcal{T}}, m) \geq \sigma_t]. \tag{53}$$

Proof. First, to simplify notation, let us write

$$\begin{aligned} V_{\mathcal{T}-t,i_{\mathcal{T}-t}}(\boldsymbol{m}^{\mathcal{T}-t},\boldsymbol{p}_{\mathcal{T}-t},\boldsymbol{p}_{-\mathcal{T}}) \equiv & \sum_{t'\in\mathcal{T}-t}\left(\mathbb{1}[p_{t'}(i_{\mathcal{T}-t},m^{t'})\geq\sigma_{t'}]\left(1+\pi_{t'|\mathcal{T}}\sum_{r\notin\mathcal{T}}\mathbb{1}[p_r(\mathcal{T}\mid t',1)\geq\sigma_r]\right)\right) \\ & + \sum_{t'\in\mathcal{T}-t}\left(\mathbb{1}[p_{t'}(i_{\mathcal{T}-t},m^{t'})<\sigma_t']\left(\pi_{t'|\mathcal{T}}\sum_{r\notin\mathcal{T}}\mathbb{1}[p_r(\mathcal{T}\mid t',0)\geq\sigma_r]\right)\right) \\ & + \left(1-\sum_{t\in\mathcal{T}}\pi_{t|\mathcal{T}}\right)\sum_{r\notin\mathcal{T}}\mathbb{I}[p_r(\mathcal{T}|\emptyset)\geq\sigma_r] \end{aligned} \tag{54}$$

$$\begin{aligned} V_{t,(i_{\mathcal{T}-t},i_{t|\mathcal{T}})}(\boldsymbol{m}^{\mathcal{T}},\boldsymbol{p}_{\mathcal{T}},\boldsymbol{p}_{-\mathcal{T}}) = & \left(\mathbb{1}[p_t((i_{\mathcal{T}-t},i_{t|\mathcal{T}}),m^t)\geq\sigma_t]\left(1+\pi_{t|\mathcal{T}}\sum_{r\notin\mathcal{T}}\mathbb{1}[p_r(\mathcal{T}\mid t,1)\geq\sigma_r]\right)\right) \\ & + \left(\mathbb{1}[p_tp_t((i_{\mathcal{T}-t},i_{t|\mathcal{T}}),m^t)<\sigma_t]\left(\pi_{t|\mathcal{T}}\sum_{r\notin\mathcal{T}}\mathbb{1}[p_r(\mathcal{T}\mid t,0)\geq\sigma_r]\right)\right) \end{aligned} \tag{55}$$

By the preceding discussion, (54) and (55) captures, respectively, the component of Sender's payoff which does not and does depend on how Sender acquires information for target t. Notice then that Sender's payoff can be written as

$$\begin{aligned} & \sum_\theta \Pr(\theta)\sum_{\boldsymbol{m}_S^{\mathcal{T}}} i^*_{\mathcal{T}}(\boldsymbol{m}_S^{\mathcal{T}}\mid\theta)\sum_{\boldsymbol{m}^{\mathcal{T}}}\left(\prod_{t\in\mathcal{T}} c^*_{t|\mathcal{T}}(m^t\mid m_S^t)\right)V_{\mathcal{T},i^*_{\mathcal{T}}}(\boldsymbol{m}^{\mathcal{T}},\boldsymbol{p}^*_{\mathcal{T}},\boldsymbol{p}^*_{-\mathcal{T}}) \\ & = \sum_\theta \Pr(\theta)\left[\sum_{\boldsymbol{m}_S^{\mathcal{T}-t}} i^*_{\mathcal{T}-t}(\boldsymbol{m}_S^{\mathcal{T}-t}\mid\theta)\sum_{\boldsymbol{m}^{\mathcal{T}-t}}\left(\prod_{t'\in\mathcal{T}-t} c^*_{t'|\mathcal{T}}(m^{t'}\mid m_S^{t'})\right)\right]\times \\ & \left(i^*_{t|\mathcal{T}}(m_S^t|\boldsymbol{m}_S^{\mathcal{T}-t},\theta)\sum_{m^t} c^*_{t|\mathcal{T}}(m^t\mid m_S^t)V_{t,(i_{\mathcal{T}-t},i_{t|\mathcal{T}})}(\boldsymbol{m}^{\mathcal{T}},\boldsymbol{p}_{\mathcal{T}},\boldsymbol{p}_{-\mathcal{T}})\right. \\ & \left. + \underbrace{V_{\mathcal{T}-t,i_{\mathcal{T}-t}}(\boldsymbol{m}^{\mathcal{T}-t},\boldsymbol{p}_{\mathcal{T}-t},\boldsymbol{p}_{-\mathcal{T}})}_{\text{Constant in } \{i^*_{t|\mathcal{T}}(\cdot|\boldsymbol{m}_S^{\mathcal{T}-t},\cdot)\}_{\boldsymbol{m}_S^{\mathcal{T}-t}\in M^{\mathcal{T}-t}}}\right) \end{aligned}$$

Hence, holding fixed how Sender acquires information for non-t targets, $i^*_{\mathcal{T}-t}$, how Sender acquires information for target t conditional on the information acquired for non-t targets, $\{i^*_{t|\mathcal{T}}(\cdot|\boldsymbol{m}_S^{\mathcal{T}-t},\theta)\}_{\boldsymbol{m}_S^{\mathcal{T}-t}\in M^{\mathcal{T}-t},\theta\in\Theta}$, must maximize her expected payoff:

$$i^*_{t|\mathcal{T}}(\cdot|\boldsymbol{m}_S^{\mathcal{T}-t},\theta) \in \underset{i_{t|\mathcal{T}}(\cdot|\boldsymbol{m}_S^{\mathcal{T}-t},\theta)\in I_t(\boldsymbol{m}_S^{\mathcal{T}-t}|i^*_{\mathcal{T}-t},\theta)}{\arg\max} \sum_{m_S^t} i_{t|\mathcal{T}}(m_S^t|\boldsymbol{m}_S^{\mathcal{T}-t},\theta)\sum_{m^t} c^*_{t|\mathcal{T}}(m^t \mid m_S^t)V_{t,(i^*_{\mathcal{T}-t},i_{t|\mathcal{T}})}((m^t,\boldsymbol{m}^{\mathcal{T}-t}),\boldsymbol{p}_{\mathcal{T}},\boldsymbol{p}_{-\mathcal{T}})$$

Because Sender's payoff is linear in probabilities, and $I_t(\boldsymbol{m}_S^{\mathcal{T}-t} \mid i^*_{\mathcal{T}-t}) = \times_{\theta\in\Theta} I_t(\boldsymbol{m}_S^{\mathcal{T}-t} \mid i^*_{\mathcal{T}-t},\theta)$, the above is equivalent to saying that $\{i^*_{t|\mathcal{T}}(\cdot|\boldsymbol{m}_S^{\mathcal{T}-t},\cdot)\}_{\boldsymbol{m}_S^{\mathcal{T}-t}\in M^{\mathcal{T}-t}}$ must solve

$$i^*_{t|\mathcal{T}}(\cdot|\boldsymbol{m}_S^{\mathcal{T}-t},\cdot) \in \underset{i_{t|\mathcal{T}}(\cdot|\boldsymbol{m}_S^{\mathcal{T}-t},\cdot)\in I_t(\boldsymbol{m}_S^{\mathcal{T}-t}|i^*_{\mathcal{T}-t})}{\arg\max} \sum_{\theta}\Pr(\theta)\sum_{m_S^t} i_{t|\mathcal{T}}(m_S^t|\boldsymbol{m}_S^{\mathcal{T}-t},\theta)\sum_{m^t} c^*_{t|\mathcal{T}}(m^t \mid m_S^t)V_{t,(i^*_{\mathcal{T}-t},i_{t|\mathcal{T}})}((m^t,\boldsymbol{m}^{\mathcal{T}-t}),\boldsymbol{p}_{\mathcal{T}},\boldsymbol{p}_{-\mathcal{T}})$$

Noting the similarity between the above and (52), it follows that to prove the weak Example Unravelling Theorem, it suffices to show that for all $\boldsymbol{m}_S^{\mathcal{T}-t} \in M^{\mathcal{T}-t}$,

$$\underset{m^t\in A_t(m_S^t)}{\arg\max}\; V_{t,(i^*_{\mathcal{T}-t},i_{t|\mathcal{T}})}((m^t,\boldsymbol{m}^{\mathcal{T}-t}),\boldsymbol{p}_{\mathcal{T}},\boldsymbol{p}_{-\mathcal{T}}) \subseteq \underset{m^t\in A_t(m_S^t)}{\arg\max}\; \mathbb{1}[p^*_t(i^*_{\mathcal{T}},m^t)\geq\sigma_t]. \tag{56}$$

The argument follows Theorem 1 in the main text, so we keep it brief. Fix any information acquired for target t, m_S^t. Suppose that regardless of the information communicated to target t, $m^t \in A_t(m_S^t)$, target t never buys the widget: $p^*_t(i^*_{\mathcal{T}},m^t) < \sigma_t$. By (55), this means that regardless of what Sender communicates to target t, the change in Sender's payoff arising for target t is the same: $V_{t,(i^*_{\mathcal{T}-t},i_{t|\mathcal{T}})}((m^t,\boldsymbol{m}^{\mathcal{T}-t}),\boldsymbol{p}_{\mathcal{T}},\boldsymbol{p}_{-\mathcal{T}})$ is constant for all $m^t \in A_t(m_S^t)$. It then follows that both sides of (56) coincide.

Next, suppose that there exists a message Sender can send to target t that convinces target t to buy the widget, i.e., $\exists m^t \in A_t(m_S^t)$ s.t. $p^*_t(i^*_{\mathcal{T}},m^t)\geq\sigma_t$. Because, by Lemma 2C, regardless of what is observed for non-t targets, non-targets buy more frequently conditional on target t buying that not buying, Sender's payoff $V_{t,(i_{\mathcal{T}-t},i_{t|\mathcal{T}})}(\boldsymbol{m}^{\mathcal{T}},\boldsymbol{p}_{\mathcal{T}},\boldsymbol{p}_{-\mathcal{T}})$ is maximized on the subset of messages in $A_t(m_S^t)$ which induce the target to buy: $p^*_t(i^*_{\mathcal{T}},m^t)\geq\sigma_t$. Thus, the LHS of (56) must be subset of the RHS of (56). □

In general, the weak Example Unravelling Theorem does not say that Sender optimally communicates to all targets as if there exists no other receiver. This is because in general, the set of information structures available to Sender, $I_{\mathcal{T}}$, may be sufficiently small so that the information Sender acquires for each receiver must always be correlated. As such, acquiring more information that raises the probability one receiver buys the widget may require lowering the probability the other receiver buys the widget, and vice versa.

Of course, the issue described above would not persist if the communication protocol is private, where Sender can freely adjust the information acquired for one target without affecting what is communicated to other targets. The next result shows that because of this, we obtain the full analogue to the single target Example Unravelling Theorem: Sender optimally communicates to all targets as if there exists no other receiver.

Theorem 2C (Strong Example-unravelling)**.** *Suppose that the communication protocol allows for pure private communication. In any equilibrium* $(\mathcal{T}^*, \{i^*_{\mathcal{T}}\}_{\mathcal{T}}, \{c^*_{\mathcal{T}}\}_{\mathcal{T}}, \{\boldsymbol{p}^*_{\mathcal{T}}, \boldsymbol{p}^*_{-\mathcal{T}}\}_{\mathcal{T}})$*, for each set of targets* $\mathcal{T}$ *and targeted receiver* $t \in \mathcal{T}$*,*

$$i^*_t \in \arg\max_{i_t \in I_{\mathcal{T}}(t)} \sum_{\theta} \Pr(\theta) \sum_{m^t_S} i_t(m^t_S \mid \theta) \sum_{m_t} c^*_t(m_t \mid m_S) \mathbb{1}[p^*_t(i^*_{\mathcal{T}}, m_t) \geq \sigma_t] \tag{57}$$

Proof. Suppose that communication is private. By (52), this means that for all $\boldsymbol{m}^{\mathcal{T}-t}_S \in M^{\mathcal{T}-t}$ drawn with strictly positive probability,

$$i^*_{t|\mathcal{T}}(\cdot|\boldsymbol{m}^{\mathcal{T}-t}_S, \theta) \in \arg\max_{i_t \in I_{\mathcal{T}}(t,\theta)} \sum_{\theta} \Pr(\theta) \sum_{m^t_S} i_t(m^t_S \mid \theta) \sum_{m_t} c^*_t(m_t \mid m_S) \mathbb{1}[p^*_t(i^*_{\mathcal{T}}, m_t) \geq \sigma_t]$$

Since this holds for each realization of $\boldsymbol{m}^{\mathcal{T}-t}_S$, it follows that aggregating over all messages $\boldsymbol{m}^{\mathcal{T}-t}_S$ drawn under $i^*_{\mathcal{T}-t}$, we have

$$\begin{aligned}
&\sum_{\theta} \Pr(\theta) \sum_{m^t_S} i^*_t(m^t_S \mid \theta) \sum_{m_t} c^*_t(m_t \mid m_S) \mathbb{1}[p^*_t(i^*_{\mathcal{T}}, m_t) \geq \sigma_t] \\
&= \sum_{\theta} \Pr(\theta) \sum_{\boldsymbol{m}^{\mathcal{T}-t}_S} i^*_{\mathcal{T}-t}(\boldsymbol{m}^{\mathcal{T}-t}_S|\theta) \left(\sum_{m^t_S} i^*_{t|\mathcal{T}}(m^t_S|\boldsymbol{m}^{\mathcal{T}-t}_S, \theta) \sum_{m_t} c^*_t(m_t \mid m_S) \mathbb{1}[p^*_t(i^*_{\mathcal{T}}, m_t) \geq \sigma_t] \right) \\
&= \sum_{\theta} \Pr(\theta) \sum_{\boldsymbol{m}^{\mathcal{T}-t}_S} i^*_{\mathcal{T}-t}(\boldsymbol{m}^{\mathcal{T}-t}_S|\theta) \max_{i_t(\cdot|\theta) \in I_{\mathcal{T}}(t,\theta)} \sum_{\theta} \Pr(\theta) \sum_{m^t_S} i_t(m^t_S \mid \theta) \sum_{m_t} c^*_t(m_t \mid m_S) \mathbb{1}[p^*_t(i^*_{\mathcal{T}}, m_t) \geq \sigma_t] \\
&= \sum_{\theta} \Pr(\theta) \max_{i_t(\cdot|\theta) \in I_{\mathcal{T}}(t,\theta)} \sum_{\theta} \Pr(\theta) \sum_{m^t_S} i_t(m^t_S \mid \theta) \sum_{m_t} c^*_t(m_t \mid m_S) \mathbb{1}[p^*_t(i^*_{\mathcal{T}}, m_t) \geq \sigma_t] \\
&= \max_{i_t \in I_{\mathcal{T}}(t)} \sum_{\theta} \Pr(\theta) \sum_{m^t_S} i_t(m^t_S \mid \theta) \sum_{m_t} c^*_t(m_t \mid m_S) \mathbb{1}[p^*_t(i^*_{\mathcal{T}}, m_t) \geq \sigma_t],
\end{aligned}$$

so (57) holds. □

D Receivers observing other receivers' actions

In this extension, Sender's objective and communication protocol are unchanged; a strategy for Sender remains a triple $(t, \{i_t\}_{t\in\mathcal{R}}, \{c_t\}_{t\in\mathcal{R}})$. The new component is that, after (possibly) observing the target, receivers truthfully announce their intended actions and, with some probability, observe others' announcements. This changes non-targets' information sets and affects their *final* (rather than merely interim) beliefs. We therefore first formalize the timing and introduce notation for interim posteriors and cross-observations $(p_r^{\text{int}}, X_j, q_j(a), W_j^r, Z_j^r)$ as well as the induced product measure $\mathbb{Q}_r$. We then adapt the equilibrium definition to this information structure and establish the analogue of the Example–Unravelling Theorem and the double-edged sword of popularity.

Timing and notation. We begin by summarizing the timing. In stage 1, Sender chooses a target t and communicates with him privately. The target then takes an action $a_t \in \{0, 1\}$. In stage 2, non-targeted receivers observe the target's action independently with probability π_t. In stage 3 all receivers (including the target) make a truthful announcement of their intention to buy or not buy. Stage 3 announcements are observed independently by each receiver with some probability which we formalize shortly. In stage 4 each non-target updates their interim posteriors to a "final posterior" and chooses whether to buy. We now fix the notation for the new objects arising particularly in stages 3-4.

- For each non-target $r \neq t$, after stage 2 the interim posterior is

  $$p_r^{\text{int}}(o) \;\equiv\; p_r(t \mid o_t^r = o), \qquad o \in \{1, 0, \emptyset\}.$$

  By Lemma 1, in any equilibrium (which we define below), $p_r^{\text{int}}(1) \geq p_r^{\text{int}}(0)$.

- In stage 3 every receiver $j \in \mathcal{R}$ announces

  $$X_j \;=\; \mathbb{1}\big[p_j^{\text{int}}(o) \geq \sigma_j\big] \in \{0, 1\}.$$

  For each non-target $j \neq t$, we define

  $$q_j(a) \;\equiv\; \Pr[X_j = 1 \mid a_t = a] \;=\; \pi_t\, \mathbb{1}\big[p_j^{\text{int}}(a) \geq \sigma_j\big] + (1 - \pi_t)\, \mathbb{1}\big[\mu \geq \sigma_j\big],$$

  and Lemma 2 implies that in any equilibrium, $q_j(1) \geq q_j(0)$. For completeness, set $q_t(1) = 1$ and $q_t(0) = 0$ since $X_t = a_t$ by truthfulness and the target's threshold rule.

- In stage 3, for each ordered pair (r, j) with $r \neq j$, receiver r observes X_j with probability $o_r(j) \in [0, 1]$, independently across pairs (and independently of stage 2). Let $W_j^r \in \{0, 1\}$ be the indicator that r observes j's announcement, so $\Pr(W_j^r = 1) = o_r(j)$, with independence across (r, j) and from earlier stages. Define

$$Z_j^r \equiv W_j^r \cdot X_j \in \{0, 1\}$$

to be an indicator for the event that r both observes j and sees $X_j = 1$. Conditioning on the realized target action $a_t = a$,

$$\begin{aligned}\Pr[Z_j^r = 1 \mid a_t = a] &= \Pr(W_j^r = 1,\ X_j = 1 \mid a_t = a) \\ &= \Pr(W_j^r = 1)\ \Pr(X_j = 1 \mid a_t = a) \\ &= o_r(j)\, q_j(a),\end{aligned}$$

and similarly $\Pr[Z_j^r = 0 \mid a_t = a] = 1 - o_r(j)\, q_j(a)$. It is straightforward to show that $\{Z_j^r\}_{j \neq r}$ are conditionally independent given a_t. Intuitively, once a_t is fixed, Z_j^r only depends on agents' observations of the target's action, which are independent.

Write $\mathbf{Z}^r \equiv (Z_j^r)_{j \neq r} \in \{0, 1\}^{R-1}$ and define the induced product measure

$$\mathbb{Q}_r(\mathbf{z} \mid a_t = a) \ \equiv\ \prod_{j \neq r} \big(o_r(j)\, q_j(a)\big)^{z_j} \big(1 - o_r(j)\, q_j(a)\big)^{1 - z_j}. \tag{58}$$

- In Stage 4, each non-target $r \neq t$ observes $S_r = (o_t^r, \mathbf{Z}^r)$ and forms the final posterior $p_r(S_r)$, then buys iff $p_r(S_r) \geq \sigma_r$.

Sender's expected payoff with cross-observation. Given a target t, a message m_t to t, and belief functions $(p_t, \boldsymbol{p}_{-t})$, define

$$\begin{aligned}V_t^{\text{obs}}(m_t, p_t, \boldsymbol{p}_{-t}) \equiv \mathbb{1}[p_t(i_t, m_t) \geq \sigma_t] \Bigg(1 + \sum_{r \neq t} \Big(&\pi_t\, \mathbb{1}[p_r^{\text{int}}(1) \geq \sigma_r] \\ &+ (1 - \pi_t) \sum_{\mathbf{z}} \mathbb{1}[p_r(\emptyset, \mathbf{z}) \geq \sigma_r]\ \mathbb{Q}_r(\mathbf{z} \mid 1)\Big)\Bigg) \\ + \mathbb{1}[p_t(i_t, m_t) < \sigma_t] \Bigg(\sum_{r \neq t} \Big(&\pi_t\, \mathbb{1}[p_r^{\text{int}}(0) \geq \sigma_r] \\ &+ (1 - \pi_t) \sum_{\mathbf{z}} \mathbb{1}[p_r(\emptyset, \mathbf{z}) \geq \sigma_r]\ \mathbb{Q}_r(\mathbf{z} \mid 0)\Big)\Bigg).\end{aligned} \tag{59}$$

This is the expected number of buyers (including t if the target buys), where for non-targets we combine the stage 2 probability (π_t) of observing t and the stage 3 probability of observing other receivers having not observed t.

Equilibrium. We preserve the baseline definition of equilibrium but augment it to capture rationality of both the interim and final beliefs of non-targets. Formally, an *assessment* is a tuple

$$\big(t, \{i_t\}_{t\in\mathcal{R}}, \{c_t\}_{t\in\mathcal{R}}, \{p_t, \boldsymbol{p}_{-t}\}_{t\in\mathcal{R}}\big),$$

where for each t,

- $p_t = p_t(i_t, m_t)$ is exactly as in the baseline model, and
- $\boldsymbol{p}_{-t}$ collects, for every non-target $r \neq t$, both an *interim* belief map $p_r^{\text{int}} : \{0, 1, \emptyset\} \to [0, 1]$ and a *final* belief map $p_r : \{0, 1, \emptyset\} \times \{0, 1\}^{R-1} \to [0, 1]$.[22]

An assessment is an equilibrium if, for every $t \in \mathcal{R}$:

1. For each $r \neq t$, $p_r^{\text{int}}(o)$ is derived by Bayes' rule from (i_t, c_t) and the target's threshold rule exactly as in the baseline (1)–(3).
2. p_t is derived as in the baseline (4)–(5).
3. For each $r \neq t$,
   - If $o_t^r \in \{0, 1\}$, then observing $\mathbf{Z}^r$ carries no additional information about θ beyond $a_t = o_t^r$. Indeed,

$$\begin{aligned} p_r(o_t^r, \mathbf{z}) &= \frac{\Pr(\theta = G, a_t = o_t^r, \mathbf{Z}^r = \mathbf{z})}{\sum_{\theta'} \Pr(\theta', a_t = o_t^r, \mathbf{Z}^r = \mathbf{z})} = \frac{\Pr(\theta = G, a_t = o_t^r)\, \mathbb{Q}_r(\mathbf{z} \mid o_t^r)}{\sum_{\theta'} \Pr(\theta', a_t = o_t^r)\, \mathbb{Q}_r(\mathbf{z} \mid o_t^r)} \\ &= \frac{\Pr(\theta = G, a_t = o_t^r)}{\sum_{\theta'} \Pr(\theta', a_t = o_t^r)} \;=\; p_r^{\text{int}}(o_t^r), \end{aligned}$$

     since $\mathbb{Q}_r(\cdot \mid a)$ does not depend on θ once $a_t = a$ is fixed.
   - If $o_t^r = \emptyset$, then by Bayes' rule

$$p_r(\emptyset, \mathbf{z}) \;=\; \frac{\mu\big(\Pr(a_t = 1 \mid G)\, \mathbb{Q}_r(\mathbf{z} \mid 1) + \Pr(a_t = 0 \mid G)\, \mathbb{Q}_r(\mathbf{z} \mid 0)\big)}{\sum_{\theta\in\{G,B\}} \Pr(\theta)\big(\Pr(a_t = 1 \mid \theta)\, \mathbb{Q}_r(\mathbf{z} \mid 1) + \Pr(a_t = 0 \mid \theta)\, \mathbb{Q}_r(\mathbf{z} \mid 0)\big)}. \tag{60}$$

[22] In a slight abuse of notation we continue to write p_r as in the baseline model, but we now view it as a function of the full signal $S_r = (o_t^r, \mathbf{Z}^r)$.

4. Given $(p_t, \boldsymbol{p}_{-t})$, Sender's choice of i_t and c_t solve

$$i_t \in \arg\max_{i \in I} \sum_{\theta} \Pr(\theta) \sum_{m_S} i(m_S \mid \theta) \sum_{m_t \in A(m_S)} c_t(m_t \mid m_S)\, V_t^{\text{obs}}(m_t, p_t, \boldsymbol{p}_{-t}), \tag{61}$$

$$c_t(m_t \mid m_S) > 0 \;\Rightarrow\; m_t \in \arg\max_{m \in A(m_S)} V_t^{\text{obs}}(m, p_t, \boldsymbol{p}_{-t}). \tag{62}$$

Note that these expression are identical to Equations (6) and (7) but with V_t^{obs} in place of V_t.

5. Sender's choice of t is sequentially optimal exactly as in (8), evaluating continuation payoffs via V^{obs}.

We gather two facts about the stage 3 announcements and stage 4 beliefs that we will use below. Equip $\{0,1\}^{R-1}$ with the product (coordinate-wise) order, so $\mathbf{y} \geq \mathbf{z}$ iff $y_i \geq z_i$ for each $i \in \{1, \ldots, R-1\}$. For two random vectors $\mathbf{Z}^{(1)}$ and $\mathbf{Z}^{(0)}$ with probability measures ν_1, ν_0 on $\{0,1\}^{R-1}$, ν_1 first-order stochastically dominates ν_0 (denoted $\nu_1 \succeq_{\text{FOSD}} \nu_0$) if

$$\Pr\left(\mathbf{Z}^{(1)} \geq \mathbf{z}\right) \;\geq\; \Pr\left(\mathbf{Z}^{(0)} \geq \mathbf{z}\right) \qquad \text{for all } \mathbf{z} \in \{0,1\}^{R-1}.$$

Equivalently, writing $U_{\mathbf{z}} := \{\mathbf{x} : \mathbf{x} \geq \mathbf{z}\}$, the FOSD condition is $\nu_1(U_{\mathbf{z}}) \geq \nu_0(U_{\mathbf{z}})$ for all $\mathbf{z}$. In what follows, $\mathbb{Q}_r(\cdot \mid a)$ denotes the product-Bernoulli *law* defined in (58). To avoid ambiguity, we write $\mathbb{Q}_r(\mathbf{z} \mid a)$ for its point mass at $\mathbf{z}$ and, for any set $A \subseteq \{0,1\}^{R-1}$, $\mathbb{Q}_r(A \mid a) \equiv \sum_{\mathbf{x} \in A} \mathbb{Q}_r(\mathbf{x} \mid a)$.

Lemma 16 (FOSD shift in observed positive announcements)**.** *Fix any equilibrium, a target t, and a non-target $r \neq t$. Then $\mathbb{Q}_r(\cdot \mid a)$ (as in (58)) satisfies,*

$$\mathbb{Q}_r(\cdot \mid a_t = 1) \;\succeq_{\text{FOSD}}\; \mathbb{Q}_r(\cdot \mid a_t = 0).$$

Proof. By (58), under $a_t = a$ the vector $\mathbf{Z}^r$ has independent Bernoulli coordinates with means $\{o_r(j) q_j(a)\}_{j \neq r}$. Construct on a common probability space i.i.d. $U_j \sim \text{Unif}[0,1]$ for $j \neq r$ and define, for $a \in \{0,1\}$,

$$Z_j^{(a)} \;\equiv\; \mathbb{1}\big\{U_j \leq o_r(j) q_j(a)\big\}, \qquad \mathbf{Z}^{(a)} = (Z_j^{(a)})_{j \neq r}.$$

Then $\mathbf{Z}^{(a)}$ has law $\mathbb{Q}_r(\cdot \mid a)$. Since $q_j(1) \geq q_j(0)$ for every j, we have $Z_j^{(1)} \geq Z_j^{(0)}$ and hence $\mathbf{Z}^{(1)} \geq \mathbf{Z}^{(0)}$ coordinate-wise. Therefore, for every $\mathbf{z} \in \{0,1\}^{R-1}$,

$$\Pr\big(\mathbf{Z}^{(1)} \geq \mathbf{z}\big) \;\geq\; \Pr\big(\mathbf{Z}^{(0)} \geq \mathbf{z}\big),$$

which is exactly $\mathbb{Q}_r(\cdot \mid 1) \succeq_{\text{FOSD}} \mathbb{Q}_r(\cdot \mid 0)$. $\square$

Lemma 17 (Monotone updating from announcements). *Fix any equilibrium, a target t, and anon-target $r \neq t$. If $o_t^r \in \{0,1\}$ then $p_r(o_t^r, \mathbf{Z}^r) = p_r^{\text{int}}(o_t^r)$. If $o_t^r = \emptyset$, then $p_r(\emptyset, \mathbf{z})$ is (weakly) increasing in each coordinate of $\mathbf{z}$.*

Proof. If $o_t^r \in \{0,1\}$, the claim is exactly item 3 in the equilibrium definition (stage 4 carries no additional information about θ conditional on a_t).

When $o_t^r = \emptyset$, write the posterior likelihood ratio as

$$\begin{aligned}
\frac{p_r(\emptyset, \mathbf{z})}{1 - p_r(\emptyset, \mathbf{z})} &= \frac{\mu}{1-\mu} \cdot \frac{\Pr(a_t = 1 \mid G)\, \mathbb{Q}_r(\mathbf{z} \mid 1) + \Pr(a_t = 0 \mid G)\, \mathbb{Q}_r(\mathbf{z} \mid 0)}{\Pr(a_t = 1 \mid B)\, \mathbb{Q}_r(\mathbf{z} \mid 1) + \Pr(a_t = 0 \mid B)\, \mathbb{Q}_r(\mathbf{z} \mid 0)} \\
&= \frac{\mu}{1-\mu} \cdot \frac{\Pr(a_t = 1 \mid G)\, R(\mathbf{z}) + \Pr(a_t = 0 \mid G)}{\Pr(a_t = 1 \mid B)\, R(\mathbf{z}) + \Pr(a_t = 0 \mid B)},
\end{aligned}$$

where $R(\mathbf{z}) \equiv \mathbb{Q}_r(\mathbf{z} \mid 1)/\mathbb{Q}_r(\mathbf{z} \mid 0)$ (interpreted as $+\infty$ if the denominator is zero and the numerator positive). Let

$$f(R) \;\equiv\; \frac{\Pr(a_t = 1 \mid G)R + \Pr(a_t = 0 \mid G)}{\Pr(a_t = 1 \mid B)R + \Pr(a_t = 0 \mid B)}.$$

Then

$$f'(R) = \frac{\Pr(a_t = 1 \mid G) - \Pr(a_t = 1 \mid B)}{\big(\Pr(a_t = 1 \mid B)R + \Pr(a_t = 0 \mid B)\big)^2} \;\geq\; 0,$$

so f is (weakly) increasing in R. Hence it suffices to show that $R(\mathbf{z})$ is (weakly) increasing in each coordinate of $\mathbf{z}$.

Fix $j \neq r$ and let $\mathbf{z}^{+j}$ be $\mathbf{z}$ with the j-th coordinate flipped from 0 to 1. From (58),

$$\frac{R(\mathbf{z}^{+j})}{R(\mathbf{z})} = \frac{\frac{o_r(j)\, q_j(1)}{1 - o_r(j)\, q_j(1)}}{\frac{o_r(j)\, q_j(0)}{1 - o_r(j)\, q_j(0)}} = \frac{\phi(o_r(j) q_j(1))}{\phi(o_r(j) q_j(0))} \;\geq\; 1,$$

where $\phi(x) := x/(1-x)$ is increasing on $[0,1)$ and we adopt the convention $\phi(1) = +\infty$. If $o_r(j) \in (0,1]$ and $q_j(1), q_j(0) \in [0,1)$ then the inequality immediately follows from the fact that $q_j(1) \geq q_j(0)$. If $o_r(j) = 0$, or if $q_j(1) = q_j(0) = 0$ then the j-th coordinate is deterministically 0 under both laws and the monotonicity is vacuous. If $q_j(0) = 0 < q_j(1)$, then the ratio is $+\infty$ so again the inequality holds. Iterating this single-coordinate argument yields that $R(\mathbf{z})$ is (weakly) increasing in each coordinate, hence so is $p_r(\emptyset, \mathbf{z})$. $\square$

We now use Lemmas 16 and 17 to show that examples (still) compel.

Lemma 2D (Examples compel)**.** *In any equilibrium* $\big(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}}\big)$*, and for each receiver* $t \in \mathcal{R}$*, if Sender targets* t*, then receiver* r*'s probability of buying the widget is greater if the target buys the widget: for all* $r \neq t$*,*

$$\Pr(a_r = 1 \mid a_t = 1) \;\geq\; \Pr(a_r = 1 \mid a_t = 0).$$

Proof. Let $a_t = a$ be the realized action of the target. Then we may decompose $\Pr(a_r = 1 \mid a_t = 1)$ by whether r observes a_t:

$$\Pr(a_r = 1 \mid a_t = a) = \pi_t\, \mathbb{1}[p_r^{\text{int}}(a) \geq \sigma_r] + (1-\pi_t) \sum_{\mathbf{z}} \mathbb{1}[p_r(\emptyset, \mathbf{z}) \geq \sigma_r]\, \mathbb{Q}_r(\mathbf{z} \mid a).$$

The first term is weakly larger at $a = 1$ because $p_r^{\text{int}}(1) \geq p_r^{\text{int}}(0)$. For the second term—which can be written as $(1-\pi_t)\mathbb{E}_{Q_r(\cdot|a)}\big[\mathbb{1}[p_r(\emptyset, \mathbf{z}) \geq \sigma_r]\big]$—Lemma 17 implies that the indicator is (weakly) increasing in $\mathbf{z}$, while Lemma 16 says $\mathbb{Q}_r(\cdot \mid 1)$ first-order stochastically dominates $\mathbb{Q}_r(\cdot \mid 0)$. It follows that the expectation is weakly larger at $a = 1$ than at $a = 0$. Hence $\Pr(a_r = 1 \mid a_t = 1) \geq \Pr(a_r = 1 \mid a_t = 0)$. □

Example Unravelling Theorem. Fix (i_t, c_t). For any allowable $m_t \in A(m_S)$, define the induced target action

$$a_t(m_t) \;\equiv\; \mathbb{1}\big[p_t(i_t, m_t) \geq \sigma_t\big] \in \{0, 1\}.$$

Let the *continuation response* of non-targets to a realized target action $a \in \{0,1\}$ be

$$G_t(a) \;\equiv\; \sum_{r\neq t} \Pr(a_r = 1 \mid a_t = a).$$

By construction, $G_t(a)$ depends on the equilibrium beliefs $(p_t, \boldsymbol{p}_{-t})$ and primitives $(\pi_t, o_r(\cdot))$, but—crucially—*does not* depend on which particular message m_t induced the action a. Lemma 2D implies

$$G_t(1) \;\geq\; G_t(0). \tag{63}$$

With this notation, Sender's expected payoff from sending m_t can be written succinctly as

$$V_t^{\text{obs}}(m_t, p_t, \boldsymbol{p}_{-t}) := a_t(m_t) \cdot \big(1 + G_t(1)\big) + (1 - a_t(m_t)) \cdot G_t(0). \tag{64}$$

Theorem 1D (Example–unravelling)**.** *In any equilibrium*

$\big(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}}\big)$, *and for each* $t \in \mathcal{R}$,

$$i_t^* \in \arg\max_{i\in I} \sum_{\theta} \Pr(\theta) \sum_{m_S\in M} i(m_S \mid \theta) \sum_{m_t\in A(m_S)} c_t^*(m_t \mid m_S)\, \mathbb{1}\big[p_t^*(i, m_t) \geq \sigma_t\big], \tag{65}$$

and, for each $m_S \in \operatorname{supp}(i_t^*(\cdot \mid \theta))$ *and each* $m_t \in A(m_S)$,

$$c_t^*(m_t \mid m_S) > 0 \;\Rightarrow\; m_t \in \arg\max_{m\in A(m_S)} \mathbb{1}\big[p_t^*(i_t^*, m) \geq \sigma_t\big]. \tag{66}$$

Proof. As in the proof of Theorem 1, it suffices to show that for any $t \in \mathcal{R}$,

$$\arg\max_{m\in A(m_S)} V_t^{\text{obs}}(m_t, p_t^*, \boldsymbol{p}_{-t}^*) \subseteq \arg\max_{m\in A(m_S)} \mathbb{1}[p_t^*(i_t^*, m) \geq \sigma_t] = \arg\max_{m\in A(m_S)} a_t(m). \tag{67}$$

First, suppose that every allowable message induces $a_t = 0$. Then the target never buys and Sender's value function is identical for all messages, so the theorem follows trivially. Hence suppose there exists an allowable message $\overline{m}_t \in A(m_S)$ with $a_t(\overline{m}_t) = 1$. For any $m_t \in A(m_S)$, subtracting (64) evaluated at $\overline{m}_t$ from (64) evaluated at some $m_t \in \arg\max_{m\in A(m_S)} V_t^{\text{obs}}(m, p_t^*, \boldsymbol{p}_{-t}^*)$ yields

$$\begin{aligned} V_t^{\text{obs}}(m_t, p_t^*, \boldsymbol{p}_{-t}^*) - V_t^{\text{obs}}(\overline{m}_t, p_t^*, \boldsymbol{p}_{-t}^*) &= \big(a_t(m_t) - 1\big)\big(1 + G_t(1)\big) + (1 - a_t(m_t)) \cdot G_t(0) \\ &= -\big(1 - a_t(m_t)\big)\big(1 + G_t(1) - G_t(0)\big) \end{aligned}$$

There are two cases to consider:

- If $a_t(m_t) = 1$, the difference is 0.
- If $a_t(m_t) = 0$, the difference equals $-(1 + G_t(1) - G_t(0)) \leq -1$ by (63).

Hence any message m_t with $a_t(m_t) = 0$ is strictly worse than $\overline{m}_t$, while any m_t with $a_t(m_t) = 1$ is a (weak) tie with $\overline{m}_t$. Therefore every *optimal* message must satisfy $a_t(m_t) = 1$, i.e. $m_t \in \arg\max_{m\in A(m_S)} a_t(m)$, which completes the proof. □

The sharpened double-edged sword of popularity. Just as in the baseline model, Sender's equilibrium payoff from targeting a receiver can be decomposed into the cases where the target does versus does not buy. The difference here is that, in addition to direct observation of the target, non-targets may observe others' truthful announcements, so the continuation values become functions of the target's popularity via the cross-observation channel.

Let

$$P_t^* \;\equiv\; \sum_\theta \Pr(\theta) \sum_{m_S} i_t^*(m_S \mid \theta) \sum_{m_t \in A(m_S)} c_t^*(m_t \mid m_S)\, \mathbb{1}\big[p_t^*(i_t^*, m_t) \geq \sigma_t\big] \tag{68}$$

be the equilibrium probability that the target buys under (i_t^*, c_t^*). Recall the continuation response

$$G_t(a) \;\equiv\; \sum_{r \neq t} \Pr(a_r = 1 \mid a_t = a) = \sum_{r \neq t} \Big[\pi_t\, \mathbb{1}[p_r^{\text{int}}(a) \geq \sigma_r] + (1-\pi_t) \sum_{\mathbf{z}} \mathbb{1}[p_r(\emptyset, \mathbf{z}) \geq \sigma_r] \mathbb{Q}_r(\mathbf{z} \mid a)\Big]$$

and write $G_t(a, \pi_t)$ to emphasize its dependence on π_t, and $G_t^*(a, \pi_t)$ for its equilibrium value.

Lemma 19 (Value decomposition with cross-observation). *In any equilibrium* $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*,*

$$\mathbb{E}\big[V_t^{\text{obs}}(i_t^*, p_t^*, \boldsymbol{p}_{-t}^*)\big] \;=\; P_t^*\big(1 + G_t^*(1, \pi_t)\big) \;+\; (1 - P_t^*)\, G_t^*(0, \pi_t). \tag{69}$$

Moreover, by Theorem 1D, P_t^* is independent of π_t.

Proof. By (64), $V_t^{\text{obs}}(m_t, p_t^*, \boldsymbol{p}_{-t}^*) = a_t(m_t)\,(1 + G_t^*(1, \pi_t)) + (1 - a_t(m_t))\, G_t^*(0, \pi_t)$. Taking the expectation over (i_t^*, c_t^*) yields (69) with weights $\Pr(a_t = 1) = P_t^*$ and $\Pr(a_t = 0) = 1 - P_t^*$. The independence of P_t^* from π_t follows from Theorem 1D, which shows that i_t^* maximizes the threshold-crossing probability and this optimization does not involve π_t. □

The representation (69) is the exact analogue of the expression in (16)— there, the continuation terms reduce to

$$G_t^*(1, \pi_t) = F_t + \pi_t\, G_t^*, \qquad G_t^*(0, \pi_t) = F_t - \pi_t\, L_t^*,$$

so that $\mathbb{E}[V_t] = P_t^*(1 + \pi_t G_t^*) - (1 - P_t^*)\pi_t L_t^* + F_t$. In the present extension, $G_t^*(1, \pi_t)$ and $G_t^*(0, \pi_t)$ are generally nonlinear in π_t because $q_j(a)$ and $\mathbb{Q}_r(\cdot \mid a)$ depend on π_t.

Popularity remains a double-edged sword and is in fact *sharpened* by the additional announcements. To see this, note that

$$q_j(a) \;=\; \pi_t\, \mathbb{1}[p_j^{\text{int}}(a) \geq \sigma_j] \;+\; (1 - \pi_t)\, \mathbb{1}[\mu \geq \sigma_j],$$

so the fact that $p_j^{\text{int}}(1) > \mu$ and $p_j^{\text{int}}(0) < \mu$ implies $\partial q_j(1)/\partial \pi_t \geq 0$ and $\partial q_j(0)/\partial \pi_t \leq 0$. Since the expected coordinates of $\mathbf{Z}^r$ are increasing in $q_j(a)$, and the beliefs $p_r(\emptyset, \mathbf{z})$ are increasing coordinate-wise in $\mathbf{z}$ (Lemma 17) the FOSD relation (Lemma 16) implies that the stage 4 expectations

$$\sum_{\mathbf{z}} \mathbb{1}[p_r(\emptyset, \mathbf{z}) \geq \sigma_r]\, \mathbb{Q}_r(\mathbf{z} \mid 1) \quad \text{and} \quad \sum_{\mathbf{z}} \mathbb{1}[p_r(\emptyset, \mathbf{z}) \geq \sigma_r]\, \mathbb{Q}_r(\mathbf{z} \mid 0)$$

move weakly in opposite directions as π_t increases (upward for $a_t = 1$, downward for $a_t = 0$). Hence, holding P_t^* fixed, the continuation term in (69) increases with π_t when $a_t = 1$ and decreases with π_t when $a_t = 0$. Intuitively, popularity now amplifies the target's example through *two* channels: (i) the baseline re-weighting of the direct-observation branch and (ii) an additional FOSD shift in the distribution over observations of other receivers.

In this precise sense, the double-edged sword of popularity from the baseline model persists and is sharpened when receivers can observe others' announcements.

E Microfounding Popularity

Suppose receivers are positioned in a connected, undirected network $G = (V, E)$, where $V = \mathcal{R}$ are the vertices and $E \subseteq 2^{\binom{V}{2}}$ are the edges. The structure of G is common knowledge to Sender and receivers. Write $N(i) = \{j \in V : ij \in E\}$ for the set of *neighbors* of i. Each edge independently transmits any neighbor's action with exogenous probability $q \in (0, 1)$ (link failure with probability $1 - q$). The diffusion process unfolds in discrete stages $s = 1, 2, \ldots, R$.[23] We write $o_r(s) \in \{0, 1, \emptyset\}$ for the action of agent r in stage s, where $\emptyset$ denotes no action.

At stage 1, the target t chooses $o_t(1) \in \{0, 1\}$ if and only if her posterior $p_t \geq \sigma_t$. For each subsequent stage $s > 1$, any node j that has acted in stage $s - 1$ broadcasts that decision to each neighbor $i \in N(j)$ with probability q. A receiver i who has not yet acted ($o_i(k) = \emptyset$ for all $k < s$) and who observes at least one neighbor's action at stage s forms a posterior from the observed neighbor(s) actions and then makes a once-and-for-all decision $o_i(s) \in \{0, 1\}$;[24] those who do not observe the action of any neighbor remain inactive ($o_i(s) = \emptyset$).

Popularity in the network model is the probability that a randomly chosen non–target—say, j—eventually sees (through some chain of neighbor-to-neighbor transmissions) an action that *originates* with the target t. This is the equivalent to the probability that t and j are connected in the *percolated* subgraph, that is, after link-failure has occurred. With

[23] Since the network is connected, the diffusion of information takes at most R stages.

[24] This assumption avoids the kind of *strategic delay* present in, e.g., Chamley and Gale (1994).

this in mind, let $G(q) = (V, E(q))$ denote the random subgraph of G obtained by retaining every edge independently with probability q. Fixing a target t, define

$$\pi_{tj}(G, q) \;=\; \Pr\big(\, j \text{ is connected to } t \text{ in } G(q)\big).$$

The quantity $\pi_{tj}(G, q)$ is the chance that some neighbor of j takes an action which can be traced, link-by-link, back to t's initial move. If j is a neighbor of t ($d(t, j) = 1$) this is just the probability that the single edge tj transmits; if t and j are not neighbors ($d(t, j) > 1$) the information travels via intermediate nodes who learn from their own neighbors' actions.)

The popularity of t is the average of these reach probabilities across all other receivers:

$$\bar{\pi}_t(G, q) \;\equiv\; \frac{1}{R-1} \sum_{j \neq t} \pi_{tj}(G, q) \;\in (0, 1).$$

$\bar{\pi}_t(G, q)$ is therefore exactly the probability that a non-target drawn uniformly at random will ever observe a neighbor's action that ultimately descends from t's decision.

Closed-form expressions for the π_{tj}'s are available only for special networks (e.g. trees, where $\pi_{tj} = q^{d(t,j)}$), or under sparsity/small-q approximations. A practical and widely used proxy is *diffusion centrality* (Banerjee et al., 2013). Let A be the adjacency matrix of G. The diffusion centrality of node t is defined as

$$\mathrm{DC}_t(A, q) \;=\; \Big[\sum_{\ell=1}^{R} (qA)^{\ell} \mathbf{1}\Big]_t,$$

which counts the expected number of "successful" walks that start at t. Because each walk that reaches j implies that j hears a neighbor's action originating from t, diffusion centrality over-counts whenever multiple walks reach the same node; but it coincides with $(R-1)\bar{\pi}_t$ (i) on trees (no two walks to a given node share an edge) and (ii) approximately on sparse graphs or when q is small, cases in which multiple successful walks are unlikely. We may therefore use the normalized diffusion centrality $\hat{\pi}_t(A, q) \equiv \frac{\mathrm{DC}_t(A,q)}{(R-1)}$ as a proxy measure of popularity, and one can view the π_t's in our model as a reduced-form representation of the $\hat{\pi}_t(A, q)$'s in an explicitly modeled network.